Characterization of aspherical lenses
by
experimental ray tracing

by

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Abstract

Aspherical lenses are widely used in different applications of modern high quality optics, due to their good optical performance and their potential to reduce the weight and size of the optical systems. Substantial progress in the production technologies of aspherical lenses in the last decade increases the demand on accurate and reliable measurement systems for aspherical surfaces. So far, different measurement methods such as interferometry, Shack-Hartmann wavefront sensors and tactile surface profiling have been proposed. However, because of the complex surface profiles of the aspherical lenses, these methods have significant limitations concerning dynamic range and flexibility. The goal of non-contact measurement of the surfaces of the aspherical lenses without a reference object (null lens or computer generated holograms) is still not achieved.

In this thesis, the potential and the limitations of the “experimental ray tracing” (ERT) method for characterization of aspherical lenses are investigated. A new approach called “aspherical surface retrieval” (ASR) based on the ERT method is proposed. This approach allows retrieval of the aspherical surface profile of the lens from the measured slopes of the transmitted rays. In addition, simultaneous measurement of the optical performance of aspherical lenses and the aspherical surface profiles within a single test unit is targeted.

Throughout the thesis work, an ERT setup is realized and a software is developed for complete automation of the setup and analysis of the measurement data. The wavefront aberrations are determined using zonal and modal wavefront reconstruction methods. The coefficients of the Zernike polynomials that are fitted to the measurement data are used to determine the focal length. In the ASR approach, the relationship between the slopes of the transmitted rays and the aspherical surface profile is derived. Furthermore, a numerical ray tracing algorithm is written for comparison of the actual and the model aspherical lens. An optimization process minimizing the difference between the actual and the model aspherical lens is used to retrieve the aspherical surface profile. Moreover, the proof-of-principle of the ASR approach is demonstrated using simulations.
A Mach-Zehnder interferometer and a Shack-Hartmann wavefront sensor measurement setup is realized for the comparison of the measurements of wavefront aberrations of commercial aspherical lenses with the ERT setup. The measurement results using the ASR approach are verified by the measurements of different aspherical lenses by a commercial tactile surface profiler. The proposed approach based on the ERT method allowed simultaneous measurements of the optical performance and the aspherical surface profile of the lenses.
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1. Introduction

The advantages of using aspherical lenses in optical systems have been known for centuries. In the early 17th century, Rene Descartes attempted to build a new mechanical device together with a French instrument maker named Ferrier. This machine was supposed to make aspherical lenses and Descartes believed that they would vastly surpass the spherical lenses [1]. Even though this device remained unrealized, the first high quality aspherical lens was presented by Francis Smethwick to the Royal Society in 1667 [2]. From that time on, the optics world had to wait until the end of the 20th century for high volume production of precision aspherical lenses.

What was the fundamental reason that motivated Descartes to make an aspherical lens already in the 17th century? It has to be considered that it was the era of astronomical observations and telescopes were the tools to image the objects at far distances. In 1609, Galileo Galilei invented the telescope by using two handmade spherical lenses [3]. However, the images obtained by the telescopes were imperfect. Descartes proposed to use aspherical lenses instead of spherical lenses. He revealed that, by this way, it would be possible to improve the imaging quality of the telescopes [4].

The imperfections at Galileo’s telescope were originated from the spherical lenses that he used in the telescope. An ideal imaging lens affects the incoming parallel rays and redirects them to a focal point. However, as can be seen from the Fig.1.1, spherical lenses do not exactly focus the incoming rays to a single point. Instead, the point of intersection shifts along the propagation direction of the rays depending on the radial coordinate of the incoming ray. This error is known as the spherical aberration. Unlike a spherical lens, an aspherical lens can theoretically redirect the incoming rays to a common point of intersection and thereby eliminate the aberrations.

What is actually the difference between aspherical and spherical lenses? The terms “spherical” and “aspherical” describe the shapes of the surfaces of these lenses. A spherical surface is described simply by the radius of curvature of the surface. The curvature does not change along the surface. In comparison, an aspherical surface can be understood as a non-
spherical surface, because it departs from being perfectly spherical. The local curvature changes along the surface (See Fig. 1.2). Here, the deviation from the spherical surface is used to eliminate the spherical aberrations. At the same time, the deviation makes the aspherical surfaces more complex and hard to manufacture.

**Figure 1.1.** Comparison of spherical and aspherical lenses. **a)** Refracted rays by the spherical lens do not pass exactly through the focal point $F$, which result to spherical aberrations. **b)** Refracted rays pass through the focal point $F$.

**Figure 1.2.** Comparison of a spherical and an aspherical surface. A spherical surface is only described by the radius of curvature of the surface $R$ which is constant along the surface. However, in an aspherical surface, local curvature changes across the surface and the surface departs from being perfectly spherical.

In the last decade, there is a substantial progress in the aspherical lens manufacturing technologies. Nowadays, aspherical lenses are increasingly used in different applications of optics. The obvious trend is the replacement of spherical lenses with aspherical lenses [5]. The basic motivation for this change is the tendency of miniaturization of optical systems in
commercial products, as well as the improvement of the image quality of the optical systems. Due to their good optical performance, a series of spherical lenses can be replaced with a single aspherical lens. Thus, by using aspherical lenses instead of spherical lenses, the number of elements in an optical system can be reduced, which also reduces the size and the weight of the system. This makes of aspherical lenses very attractive especially for digital cameras, CD-DVD players and small digital phones which are currently in use in daily life.

Even though aspherical lenses have significant advantages over spherical ones, spherical lenses are still more frequently used. This is basically because of the unique geometry of the aspherical lenses; the manufacturing process is much more complex than the conventional spherical lenses. It requires more time and skill. Today, good quality aspherical lenses are still up to ten times more expensive than the corresponding spherical ones [6]. Therefore, there is an increasing interest on the development of cost attractive and reliable production technologies of aspherical lenses.

In general, there are two manufacturing processes used in the production of aspherical lenses; Molding and classical fabrication. This classification is basically influenced by the cost considerations and the tolerances required in the application fields.

Molded aspherical lenses are typically employed in the field of consumer electronics. The main advantage of molding techniques is the possibility of the high volume production which reduces the cost of a single produced aspherical lens and allows the competition with spherical lenses. These aspherical lenses are commonly used in the mobile phone and small digital camera industry.

Classical fabricated aspherical lenses are more used in the optical systems where high imaging quality is required. These lenses are also called as the precision aspherical lenses. They are used in different optical systems such as in high quality camera systems, zoom lenses, f-theta lenses, beam expanders and in various fields of aerospace and defence applications. Mass production of these types of aspherical lenses is currently limited because the manufacturing process is much more complex and time consuming compared to that of molded aspherical lenses. This is because that the classical fabrication is done iteratively at different steps. At each step, aspherical surface has to be measured by an appropriate measurement device and the fabrication relies on the data obtained from the measurement device. So, metrology is an important part of the production process which strongly affects the quality, the speed and the cost of the complete process.
The critical parameters of an aspherical lens measurement device are the accuracy, the resolution, the dynamic range and the measurement time. Existing measurement systems have significant limitations for in-process metrology for production and quality assurance [7]. As a consequence, there is a growing demand on alternative measurement techniques for aspherical lenses.

The scope of this work is concentrated on the metrology of the classically fabricated aspherical lenses. It should be noted that measurement systems used for testing precision lenses are also applicable for testing molded lenses. However, the fact is that the tolerances required for precision lenses are much tighter than that of molded lenses; the interest in optical metrology basically grows in the measurement techniques used to monitor the classical fabrication.

### 1.1. Aspherical lens metrology

Aspherical lens metrology can be generally classified in two categories: evaluation of individual production process errors and system level performance. The production process errors basically consist of the individual surface errors introduced during the manufacturing and the deviations of the refractive index of the delivered material. Each of these errors contributes to the overall error budget of the lens and they should be quantified at each step of the manufacturing process. Naturally, the final aim is to deliver a lens which is in the optical performance tolerance range defined by the optical designer.

A typical aspherical lens consists of two surfaces: an aspherical and a spherical or a plane surface. Surfaces are produced one after another and the sequence is choice of the manufacturer. Spherical surfaces can be fabricated traditionally with high accuracies and tested typically by an interferometer. The main difficulty occurs at the measurements of the aspherical side of the lens which is inherent at its fabrication process.

The classical fabrication process of aspheres can be done at three steps; Generating, Polishing and Finishing [8]. The complete process is typically controlled by a computer. At each step, suitable metrology is required in order to feedback the deviations from the designed surface. This process is repeated with iterative steps until deviations from the design data are in the tolerance range. The complete classical fabrication process is simply illustrated in the Fig. 1.3.
Grinding process is the first step in the classical fabrication process. At this step, the basic geometry or the form is given to the aspherical surface. According to Buchenauer [9], for ultrafine grinding process Peak to Valley (P-V) surface deviation from the design is less than 1 µm and the surface roughness is less than 200 nm. Here, an appropriate measurement system is required for testing rough aspherical surfaces.

The main task in the polishing process is to remove the roughness and the mid-frequency errors. Acceptable surface roughness for polishing process is less than 1 nm. On the other side, the maximum P-V surface deviation stays less than 1µm as in the grinding process.

There exist different surface finishing methods such as Magnetorheological Finishing [10], and Ion Beam Polishing [11]. Using these methods, it is nowadays possible to obtain P-V surface deviations in the order of tens of nanometers. But, of course, since all the systems work in an iterative manner, they all strongly depend on the quality and possibilities of metrology.

After the surface manufacturing steps are finished, as a final step, optical system performance of the lens is measured. Various parameters are quantified and checked if they are in tolerance range. In general, this process is not an iterative process as individual surface measurements. It represents the quality of the complete fabrication process.

### 1.1.1. Characterization of an aspherical surface

According to [12], a general rotationally symmetric aspherical surface represented by an equation of the form

$$Z(r) = \frac{r^2/R}{1 + \sqrt{1-(1+k)\frac{r^2}{R^2}}} + \sum_{i=2}^{n} A_{2i}r^{2i}$$  \hspace{1cm} (1.1)
where $Z$ is the surface sag, $R$ is the radius of curvature and $k$ is denoted as the conic constant. The radial distance from the optical axis $r$ is related to the coordinates $x$ and $y$ by

$$r = \sqrt{x^2 + y^2}$$

(A.2)

$A_{2i}r^{2i}$ are the higher order aspherical deformation terms. These terms represent the deviation of the surface from the conic section which is defined by the first term of the right hand side of the Eq. 1.1. Neglecting the aspherical terms, one can obtain spheres, parabolas, hyperbolas and ellipses by choosing appropriate conic constant $k$. It should be noted that the indexing of the aspherical terms starts with $i=2$, so the first aspherical term is of fourth order: $A_4r^4$. This is important, because in some representations of aspherical surfaces, indexing starts with 1, and therefore the second order coefficient $A_2$ is as well included in the aspherical equation. However, conic constant $k$ can be used to represent the second order deformation term.

Optical surface topography can be classified by three spatial regions [13]. These are the form or the geometry, waviness and the roughness. Form can be understood as the low frequency spatial region (basic aspherical shape given in Eq. 1.1), waviness as mid-frequency and roughness as high frequency. Table 1.1 shows the classification of the typical surface topography of aspherical lenses [14].

<table>
<thead>
<tr>
<th>Form (Low frequency)</th>
<th>Waviness (Mid-frequency)</th>
<th>Roughness (High frequency)</th>
</tr>
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<tr>
<td>$\Lambda \geq 1 \text{ mm}$</td>
<td>$20 \mu\text{m} \leq \Lambda \leq 1 \text{ mm}$</td>
<td>$\Lambda \leq 20 \mu\text{m}$</td>
</tr>
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</table>

The definitions of these regions are ambiguous in the literature. Especially the boundaries between the regions and the definition of the mid-frequency region are not clear. For example Aikens et.al [15] define the mid frequency spatial period between 0.5 mm and 4 mm. This ambiguity can be attributed to the difference in polishing and finishing techniques. Each method works on a different spatial region while the removal of the material from the surface. This, of course, results to different frequencies for residual surface deformations.

Specifications of the tolerances of the aspherical surfaces involve the comparison of the measurement of the manufactured surfaces and their theoretical forms. Reference [16] handles the surface form specifications and tolerances for spherical surfaces. Currently, they are as well applied for aspherical surfaces.

There is typically a series of analysis that should be done with the measured aspherical surface data. P-V deviation of actual surface shape from the theoretical or design surface shall be quantified. This can also be referred as the global deviation from the specified surface...
geometry. Root-mean-square (RMS) of the global deviation is also calculated. Furthermore, the irregularities of the surface form, which is also called as the residual surface deformations, have to be quantified. Residual surface deformations indicate the deviation of the actual surface shape from the best-fit radius aspherical shape.

Assume that the designed and the actual aspherical surfaces are denoted as \( Z_d \) and \( Z_a \) respectively. Then, the global surface deviation is simply given as

\[
\Delta_g (r) = Z_a (r) - Z_d (r)
\]  

(1.3)

Residual surface deformations are calculated first by determining the best-fit radius aspherical surface; \( Z_f (r; R_f, k_d, A_d) \). Here, \( r \) is the independent variable of the function which defines the radial distance from the optical axis. \( R_f \) is the variable radius of curvature value that is used in the fitting process and it is called as the best-fit radius value. For the conic constant and the aspherical coefficients, their design values are used and they are not variable. Then, residual surface deformations are simply calculated by

\[
\Delta_{sd} (r) = Z_a (r) - Z_f (r; R_f, k_d, A_d)
\]  

(1.4)

For illustration of the issue described above, an example aspherical surface data set is simulated and the results are shown in Fig. 1.4. P-V global deviation of the simulated aspherical shape from the theoretical aspherical shape is 2.984 µm. When the comparison is made by subtracting the best-fit radius aspherical shape, P-V surface deviation decreases to 0.730 µm. Both of these quantities are valuable for the manufacturer to analyse the quality of the manufacturing process. When one considers the global deviation shown in the Fig 1.4, it can be seen that the global deviation consists of spherical shape and higher frequency surface deformations. This is the reason why the P-V of global deviation is much greater than the P-V surface deviations after the best-fit radius aspherical surface is subtracted. For example, P-V surface deformations might be in the tolerance range, on the other side P-V global deviation can be much larger than the tolerance range. Therefore, one should be extremely careful during the analysis of these results.
Figure 1.4. Simulations of aspherical surface deviations. Global deviation $\Delta_g$ is calculated by subtracting the design surface data from the measured surface data. For surface deformations $\Delta_{sd}$, best-fit radius aspherical surface is subtracted from the measured surface data.

Aspheres can be classified by their P-V deviations from the best-fit radius aspherical surface. For example, Köhler [17] classifies the aspheres as precision, high precision and ultra precision aspheres. Table 1.2 shows the classification. It is clear that further increasing the surface quality of the aspheres, reducing the P-V deviation in the order of several nanometers, the required accuracy for the measurement systems also increases.

Table 1.2. Classification of aspheres depending on the P-V deviation from best-fit radius asphere [17]

<table>
<thead>
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<th>Precision aspheres</th>
<th>High precision aspheres</th>
<th>Ultra precision aspheres</th>
</tr>
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<tbody>
<tr>
<td>P-V</td>
<td>1000 nm</td>
<td>100 nm</td>
<td>10 nm</td>
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Even though ISO standards deliver information about the aspherical surface geometry and the tolerances, the discussion on the comparability of the measurement results from different measurement techniques still continues [18]. Different polynomial representations such as the conventional Zernike polynomials and recently proposed orthogonal based Forbes polynomials [19] are used at different measurement systems. Furthermore, mathematical procedures used in the analysis are not standardized.

In addition to the direct aspherical surface measurements, there is a growing demand on the determination of the surface slope errors during the manufacturing process of aspherical lenses [20]. Recently, new investigations are reported about the significance of the surface slope errors on the image quality of the lenses. Especially for precision aspherical lenses,
these errors can degrade the optical system performance [15]. Thus, meeting only the
tolerances of the surface form errors is not sufficient. There is also a need to measure and
specify the surface slope errors of the aspherical lenses in order to ensure that the overall
optical performance of the lens is in the tolerance range.

Surface slope error or surface tangent error can be defined as the deviation of the local surface
normal from the theoretical surface normal [6]. This parameter is defined by the optical
designer to specify the maximum allowable slope error. There is still no standard accepted for
this error in metrology. ISO10110 still does not offer an error analysis for surface slope
errors; it can be simply indicated as a text note. Fig. 1.5 depicts the basic idea of surface slope
errors.

Surface slope error is defined in a lateral region at the surface. This is called as the Integration
Length (L in Fig 1.5). Inside this region, the design and actual slopes are averaged. As there is
no standard accepted for surface slope errors, there is also no mathematical description in the
literature. In this work, the surface slope error $\Delta s$ is calculated by the formula

$$
\Delta s = \left| \frac{1}{N} \sum_{i=1}^{N} s_a(x_i) - \frac{1}{N} \sum_{i=1}^{N} s_d(x_i) \right|
$$

(1.5)

where $s_a$ and $s_d$ are the actual and the design slope values consecutively. $x_i$ is the lateral
position where the slopes are calculated and $N$ is the number of sampling inside the
Integration length which is given by

$$
N = \frac{L}{\Delta x}
$$

(1.6)
Here, $\Delta x$ is the sampling spatial resolution of the measurement. Unfortunately, there is no agreement in the literature on the units used for this error type. Different units such as degrees, radians, arcmin, waves per centimeter and waves per inch are frequently used. Like the surface form errors, both P-V and RMS surface slope errors shall be quantified during analysis of the measurement results.

1.1.2. Optical system performance

After the manufacturing process of the aspherical lens is completed, optical performance of the finished lens has to be tested. At this stage, optical performance of the produced lens is checked if it meets with tolerances defined initially by the optical designer. In general, there is no difference between testing of optical performance of aspherical lenses and conventional spherical lenses. The common parameters that describes the optical performance of a lens can listed as

- Focal length
- Wavefront aberrations
- Point spread function (PSF)
- Strehl intensity ratio
- Optical transfer function (OTF)

In general, these parameters give the information about the quality of the image of an object generated by the lens. Focal length is the well known parameter which is a paraxial property of the lens. It defines how fast the lens effects the input phase distribution of the light. This is also known as optical power of the lens. Wavefront aberrations are the most critical parameter of optical performance. It is the deviation of the actual phase distribution generated by the lens from the ideal lens.

Point Spread Function is the response of an imaging system to a point source which is the measure of two dimensional intensity distribution of point object at the paraxial (Gaussian) image plane [3]. PSF of an imaging system is given by

$$PSF = |F\{P(x,y)\}|^2$$  \hspace{1cm} (1.7)

where $P(x,y)$ is the complex pupil function and $F$ denotes the Fourier transformation,

$$P(x,y) = A(x,y) \cdot \exp \left( i \frac{2\pi}{\lambda} W(x,y) \right)$$  \hspace{1cm} (1.8)
Here, \( A \) is the amplitude and \( W(x,y) \) is the wavefront aberration function containing the phase information at the exit pupil of the lens. PSF of an ideal system is known as the Airy pattern. Fig. 1.6 illustrates an example of PSF simulations of an ideal and an aberrated system. For an ideal system, intensity is a maximum at the focal point which is located on the paraxial image plane. PSF is limited by the diffraction at the pupil of lens. However, when aberrations are present, resulting from the phase deviations represented by \( W(x,y) \), intensity distribution is spread over the different transverse locations at the paraxial image plane.

![Figure 1.6. Point spread function of (a) an ideal (unaberrated) system (Airy disk) and (b) an aberrated system.](image)

Strehl ratio is the ratio of the intensity of the Airy disk and the aberrated system at the paraxial image point of the system. Strehl ratio can be written as

\[
R_s = \frac{I_{\text{aberrated}}}{I_{\text{airy disk}}}
\]  

(1.9)

Strehl ratio of 0.8 corresponds to the well known quarter-wavelength limit of Rayleigh. Above this limit, as stated in the Marechal criterion, the optical performance of a system is regarded as well-corrected [21].

Optical Transfer Function is the frequency response, in terms of frequency, of an optical system to sinusoidal distributions of light intensity in the object plane [22]. The relationship between OTF and PSF can be given by

\[
OTF = |F\{PSF\}|^2
\]  

(1.10)

As can be seen from Eq.s 1.7 to 1.10, one common term that is very important is the information about the wavefront aberration function. Once it is measured for a lens, all parameters defined above for describing the optical system performance can be determined.
1.2. State of art – Measurement methods

As mentioned above, characterization of aspherical lenses can be handled in two categories: optical system performance and individual surface measurements. In this section, existing measurement systems will be as well discussed in two subsections as wavefront sensing methods for optical system performance testing, and aspherical surface measurements for individual surface testing.

1.2.1. Wavefront sensing methods

There are different techniques to measure the optical performances of the aspherical lenses in the literature [23]. Here, the most common three methods will be discussed.

1.2.1.1. Interferometry

Interferometry is the most common method for determination of the characteristics of optical components [24]. It is based on superposition of a high accurate reference wavefront and measured wavefront from the test device. Deviations from the reference wavefront results in interference patterns and wavefront aberration function can be determined by analyzing these patterns.

Different types of interferometers are used in optical testing such as Twyman-Green, Mach-Zehnder and Fizeau interferometer. In this work, Mach-Zehnder interferometer will be considered for measurements of wavefront aberrations. Schematic diagram of this type of configuration is illustrated in Fig. 1.7. Here, laser light is split into two arms by a beam splitter. One arm travels the path through the microscope objective and the test lens. This arm is called the test arm. At the other arm (reference arm), light travels undisturbed directed by two mirrors. Both arms are combined by a beam splitter and the resulting interference pattern is recorded by a camera. Note that the accuracy of the interferometers strongly depends on the optical components used in the measurement setup. This requires accurately calibrated reference optics.

Compared to Mach-Zehnder configuration, basic disadvantage of Twyman Green and Fizeau configuration is that the light passes two times through the lens under test. They are also called the double pass configurations. The assumption is that the optical path difference (aberrations) generated by the test lens is simply doubled due to double pass of the light. This argument leads to errors due to the fact that for large aberrations and aspherical wavefronts, the reflected light from reference mirror will not travel the same path as it passed the lens first.
time [25]. Therefore, in this work, single pass configuration is preferred for accurate transmission measurements.

\[ I(x, y, \delta) = a(x, y) + b(x, y) \cos(\phi(x, y) + \delta) \] (1.11)

where \( \phi(x, y) \) is the phase difference between the reference and the test arm at the origin and \( \delta \) is the given phase shift with respect to the origin. \( a \) is the intensity bias and \( b \) is the half of the P-V intensity modulation. The unknown phase term \( \phi(x, y) \), which holds the aberration information of the test lens, can be computed using various algorithms such as Three step, Four step, Hariharan etc. An application of this type interferometer will be presented in Chap. 4.

In the interferometric methods, monochromatic and spatial coherent light source is required for observation of fringe pattern. However, applications and design of optical components are often not restricted to one wavelength. Dynamic range of an interferometer is directly related with the resolution of the camera. The condition for the maximum measurable wavefront deviation is that the highest interference fringe frequency should be lower than the Nyquist frequency given by the pixel sample rate of the camera [27]. Therefore, strongly aberrated or aspherical wavefronts cannot be directly measured by interferometers. Interferometric tests fail in case of vibration or air turbulence in the measurement environment. Using a partially
evacuated chamber and vibration isolation devices, the shortcomings of these problems can be compensated. However, this would further increase the cost of the system.

### 1.2.1.2. Shack-Hartmann wavefront sensors

Wavefront sensors provide an alternative method for investigations of optical components. The Shack-Hartmann sensors (SHSs) are the most known wavefront sensors which are also used in ophthalmology, optical testing, laser beam characterization and adaptive optics [28].

The principle of SHSs is based on the Hartmann test screen invented in early 1900’s [29]. In his work, Hartmann tested the Great Refractor in Potsdam placing a screen with an array of holes in front of the test object. Reflected wavefront was sampled by the holes on the screen and the positions of the spots generated by each hole were observed at the detector. By calculating the transverse position deviations from the reference spot diagram, transverse aberrations were calculated. In 1971 Shack and Platt improved Hartmann’s method by implementing a lens array instead of the Hartmann screen [30]. The motivation for this replacement was to increase the intensity level of the spots at the detector. Modern SHSs consist of two basic components: a microlens array and a CCD camera. CCD camera is located at the focal plane of the microlens array. Fig. 1.8 depicts the schematic diagram of the principle of a SHS.

Incident wavefront $W$ is sampled by the microlens array which generates multiple spots, each corresponding to a microlens, on the CCD camera. Initially a reference wavefront, typically a plane wavefront, is taken in order to record the reference spot positions $x_{\text{ref}}$ at the camera.

Then, actual wavefront to be measured is launched to the sensor. Deviation in the spot location $\Delta x_{ij}$ for the corresponding microlens can be calculated by subtracting the reference spot location $x_{\text{ref}ij}$ from the measured spot location $x_{ij}$. Relationship between the deviations in the spot locations and the partial derivate of the aberrated wavefront is expressed as

$$ \begin{bmatrix} \frac{\partial W(x,y)}{\partial x} \\ \frac{\partial W(x,y)}{\partial y} \end{bmatrix}_{ij} = \frac{1}{f} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}_{ij} = \begin{bmatrix} \tan \alpha_x \\ \tan \alpha_y \end{bmatrix}_{ij} \tag{1.12}$$

where $f$ is the focal length of the microlenses. $i$ and $j$ indicate the indices of the microlens array and $\alpha$ is corresponding calculated angle. Note that the SHSs measure the deviations in the spot locations relative to the reference spot locations. Thus, accurately calibrated reference wavefronts are required.
In general, the dynamic range of SHSs can be defined as the maximum measurable wavefront deviation by the sensor. Therefore, since the SHSs measure the wavefront gradients, the dynamic range is limited by the maximum measurable gradients. However, Rocktäschel et. al. [27] described that, for aspherical or strongly aberrated wavefronts, the limitation of dynamic range is given by the maximum measurable gradient change of the wavefront. This is basically because of the fact that, if a strong gradient change occurs in the wavefront, two neighbouring spots can be overlapped. Thereupon, it is not anymore possible to assign the spot locations to the microlenses. Thus, the limitation of the dynamic range of the SHSs can be defined by the second derivative of the wavefront as [27]

$$W^* < \frac{1}{f} - c \frac{2\lambda}{d_A^2}$$  \hspace{1cm} (1.13)$$

where \(d_A\) is the diameter of a microlens and \(c\) is the factor between ideal and actual spot diameter. Here, it can be seen that a short focal length \(f\) and a large diameter \(d_A\) of microlens lead to an increase in dynamic range of the sensor. An appropriate combination of these two parameters has to be chosen for the intended dynamic range of the sensor. Pfund et. al. [31] proposed a new method based on a modified unwrapping algorithm to enhance the dynamic range of SHSs. Using this method, dynamic range can be increased by four times compared to the conventional SHS algorithms. However, maximum measurable P-V wavefront is still in the order of 100 waves.

The wavefront sensitivity can be described as the minimum detectable wavefront deviation. In SHSs, the wavefront sensitivity is related to the minimum measurable wavefront gradient by the sensor. The wavefront sensitivity can be written as [32]
\[
W_{\text{min}} = \frac{q d_4}{f} \tag{1.14}
\]

where \( q \) is the standard deviation of the determination of spot positions. Here, \( q/f \) describes the minimum measurable wavefront gradient by the sensor. Therefore, by using microlenses with larger focal lengths \( f \) and smaller diameters \( d_4 \), one can increase the wavefront sensitivity. However, as can be seen the Eqs. 1.13 and 1.14, this is the trade-off between dynamic range and wavefront sensitivity.

A typical configuration for measuring the aberrations of lenses with a SHS is illustrated in the Fig. 1.9. This configuration is also called the reverse setup. The lens under test is illuminated by a point light source which produces a spherical wavefront. Typically, laser light is coupled into a single mode fiber. Using a micrometer XYZ stage, fiber is aligned to the optical axis of the lens under test. Alternatively, a pinhole can be used to spatially filter the incident wavefront.

![Diagram](image)

**Figure 1.9.** A typical Shack-Hartmann measurement setup for testing the aberrations of lenses.

In general, apertures of the test objects are greater than the sensitive area of the CCD camera. By using a telescope (relay optics shown in Fig. 1.9), wavefront exiting the test lens is adapted to the size of the wavefront sensor aperture. Since an aberrated wavefront continuously changes its shape as it travels in space, the wavefront aberrations should be measured at the location where the instrument aberrations are represented by the wavefront distortions, hence at the exit pupil [33]. Therefore, principle plane of the lens under test is typically imaged on the microlens array of the sensor using a 4f setup (relay optics). However, uncertainty of the relay optics is the major cause of inaccurate optical testing for these sensors [34].

In SHSs, a reference optical component is not required for weak aspherical wavefronts. However, as mentioned above, if a strong gradient changes occur in the wavefront, dynamic
range of the sensor is limited by the overlapping of two neighbouring focus spots. Hence, for strong aspheric wavefronts a null lens is required [27]. The maximum number of measurement points is limited to the pixel number of the sensor, because all measured spots should be captured by the sensor at the same time. Compared to interferometric measurements, SHSs provide a more flexible and compact method, in which the sensitivity of vibrations and misalignments are substantially reduced.

1.2.1.3. Experimental ray tracing

Similar to the SHSs, the fundamentals of the “experimental ray tracing” (ERT) method goes back to the Hartmann screen tests where two photographic plates were used to detect points of intersections of the light beam at two parallel planes [35]. In 1988, Häusler et.al [36] enhanced the method by implementing a position sensitive detector (PSD) instead of the photographic plates. This was the first time that the term “experimental ray tracing” was proposed for this method. The measurement principle is transmitting a narrow laser beam through the test object and capturing the light spot at two different planes along space. The use of PSD improved the resolution of the position determination of the light spot on the detector area up to 1µm [36].

After Häusler’s work, several other investigations using the ERT method have been published. Navarro and Moreno-Barriuso [37] presented the measurements of aberrations of optical systems using the method called “Laser ray tracing”. Canabal et. al. [38] measured the refractive power of spherical lenses by the “Laser beam deflectometer”. An interesting work was published by S. Olivier [39,40], where the incoming wavefront is scanned sequentially by a programmable moving aperture using a liquid-crystal display. Moreover, Maetz [41] used ERT method to test the optical properties of the lens systems in nuclear microprobes. Each of the ERT based system has slightly varying scanning setups. The common point in all these works was not only the principle of the method, but also that the investigations were basically limited to the testing of optical properties of the test lenses.

The schematic diagram of the ERT setup is illustrated in Fig. 1.10. The main principle is to measure the slopes of the transmitted rays and thereby local gradient of the transmitted wavefront through the lens under test. A pencil test ray is physically traced through the test lens at a given \((x,y)\) position at the entrance pupil and the intensity distribution is captured by a camera behind the test lens. The camera is mounted on a translation stage which allows capturing the transmitted rays at different planes along the propagation direction \((z\text{-direction})\) of the test ray.
The centroids $S$ of the intensity distributions at the $k$th $z$-position of the camera are determined by calculating the center of mass coordinates as

$$S_x^k = \sum \frac{I_{mn}^k x_{mn}'}{\sum I_{mn}^k}, S_y^k = \sum \frac{I_{mn}^k y_{mn}'}{\sum I_{mn}^k}$$

(1.15)

where $I_{mn}^k$ is the pixel brightness given as digital number. $x_{mn}'$ and $y_{mn}'$ are the coordinates of the pixel $(m, n)$ of the camera. In general, slopes of the transmitted rays can be defined by

$$T_x = \frac{\partial S_x}{\partial z}, T_y = \frac{\partial S_y}{\partial z}$$

(1.16)

The setup that generates the test ray is also mounted on a programmable XY translation stage. This allows to scan the complete aperture of the lens under test and for each test ray position $(x, y)_i$, the slopes of the transmitted ray can be determined. Then, the transmitted wavefront through the lens can be determined from the slopes of the transmitted rays with respect to $x$ and $y$ directions as

$$\hat{V}W(x, y) = \begin{bmatrix} \frac{\partial W(x, y)}{\partial x} \\ \frac{\partial W(x, y)}{\partial y} \end{bmatrix} = \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

(1.17)

A detailed analysis of the properties and the limitations of the ERT method will be discussed in Chap. 3.
1.2.2. **Aspherical surface measurement methods**

Interferometers are superior in precision and resolution for spherical surface measurements [25]. The reflected wavefront from surface under test is combined with a reference spherical wavefront on a camera. This results to an interference pattern which contains the information about the deviation of the surface under test from the reference surface. In case of aspherical surfaces, the reflected wavefront from the surface under test has also an aspherical shape. As asphericity of the reflected wavefront increases, the frequency of the resultant interference pattern increases. When the dynamic range of interferometer (limited by the Nyquist frequency) is exceeded, interference pattern is no longer detectable. Thus, the aspherical component of the reflected wavefront shall be compensated. One common solution to this problem is the use of computer generated holograms (CGHs) [42,43]. A typical Fizeau interferometer setup using CGH for the measurements of surfaces of aspherical lenses is illustrated in Fig. 1.11.

In this measurement setup, a reference transmission sphere is utilized which reflects a portion of the incoming laser beam back to the camera. This is considered to be the reference spherical wavefront of the interferometer. The wavefront which is transmitted through the reference transmission sphere is the test wavefront. This wavefront is adapted to the aspherical part of the test surface by using a CGH which is located right behind the reference transmission sphere. The reflected wavefront from aspherical surface carries the information about the aspherical surface and passes one more time through the CGH and the reference transmission sphere. Thereby, the phase deviations between the test and the reference wavefronts from the transmission sphere can be minimized. This allows the interference pattern to be detectable at the camera. However, different types of aspheres naturally have different aspherical parts. Thus, for each asphere type, there is a need for a specific CGH which makes this method inflexible. In addition, accuracy of this method also strongly depends on the quality and alignment of the CGH [44]. Furthermore, CGHs are costly and production durations are long. This is problematic especially for the manufacturers offering customer oriented solutions.
Recently, Küchel proposed [45] and developed [46] a new measurement technique for aspherical surfaces based on Fizeau interferometry. In this technique, test asphere is scanned along its optical axis. Measurements are done at zones of normal incidence with respect to a reference spherical wavefront. By scanning the asphere, dynamic range of the interferometer is increased up to 800 µm deviation from the spherical shape and there is no need for CGHs. However, it still depends on the accuracy of the reference surface, moreover, calibration and measurement system requires expensive components.

In surface metrology, interferometric methods require high reflectivity from surface under test. Thus, they are not suitable at the grinding stage of the classical fabrication where the surface of lens is rough. They can be only used at the control processes of polishing and finishing.

SHSs are also very attractive for aspherical surface metrology [47]. Especially their capability of very fast measurements makes this method very attractive for in-process metrology for high volume production of aspherical lenses. However, poor resolution of SHSs is a significant limitation. Moreover, as mentioned in the previous section, for strong aspheres a null lens or a CGH is required.

A straightforward method for measuring aspherical surfaces is the use of surface profilers. The basic principle is to touch the aspherical surface with a stylus tip and to measure the deviations of the position of the stylus tip while scanning the surface under test. Recently, Beutler [48] reported a new method combining a profilometer with a rotational measuring axis for aspherical surface measurements. The advantages of surfaces profilers are the high dynamic range and the flexibility. They can be also implemented in the grinding process, since surface roughness is not a problem. The most important disadvantage of the surface profilers comes from the principle of the method: physical contact on the sample surface. This
can have severe impacts on the surface quality and thus the performance of the precision aspherical lenses.

1.3. Motivation and Aim

The basic motivation of this thesis is the growing demand in the aspherical lens manufacturing for accurate and reliable measurement instruments. The limitations and various disadvantages of the existing methods encourage new investigations. A summary of the properties of the above discussed measurement methods are given in Table 1.3. As a consequence, the goal of non-contact measurement of aspherical lenses without a reference object (Null lens or CGH) is still not achieved.

The aim of this PhD thesis is to investigate the potential of the experimental ray tracing method for characterization of aspherical lenses. Based on the principles of the experimental ray tracing, a new approach called aspherical surface retrieval (ASR) which enables to determine the aspherical surface profiles from the slopes of the transmitted rays is proposed. Thereby, for the first time, simultaneous measurement of the optical performance of aspherical lenses and the aspherical surface profiles within a single test unit is targeted.

Table 1.3. A summary of the properties of the existing measurement techniques for aspherical lens metrology.

<table>
<thead>
<tr>
<th></th>
<th>Interferometry</th>
<th>SIHS</th>
<th>Surface profilers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement method</td>
<td>Optical</td>
<td>Optical</td>
<td>Tactile (Contact on surface)</td>
</tr>
<tr>
<td>Optical performance</td>
<td>requires a different measurement setup</td>
<td>requires a different measurement setup</td>
<td>not possible</td>
</tr>
<tr>
<td>measurements</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resolution</td>
<td>High</td>
<td>Average</td>
<td>depends on number of scanning points</td>
</tr>
<tr>
<td>Dynamic Range</td>
<td>limited by Nyquist frequency</td>
<td>limited by overlapping of neighbouring spots</td>
<td>High (limited to extremely steep surfaces)</td>
</tr>
<tr>
<td>Reference object</td>
<td>Null and/or CGH</td>
<td>Null and/or CGH</td>
<td>No requirement</td>
</tr>
<tr>
<td>Flexibility</td>
<td>requires reference objects for every other lens</td>
<td>requires reference objects for every other lens</td>
<td>High</td>
</tr>
<tr>
<td>Measurement speed</td>
<td>High</td>
<td>Extremely high</td>
<td>depends on scanning speed</td>
</tr>
<tr>
<td>Setup time</td>
<td>Long (Preparation of CGH)</td>
<td>Long (Preparation of CGH)</td>
<td>Short</td>
</tr>
</tbody>
</table>
1.4. Outline of the thesis

This thesis consists of four chapters. In the second chapter of this thesis, first, the basic theory of the experimental ray tracing method will be reviewed. Special attention is given to the wave and ray aberrations and representation of wavefront aberrations using Zernike polynomials. Reconstruction of the wavefront from local slopes of the wavefront using the modal and zonal methods will be described. A method for determination of the focal length of lenses using the Zernike polynomials will be introduced. Furthermore, the problems of polynomial fitting to the gradient of Zernike polynomials will be discussed and a solution using an orthogonalization method will be presented. Then, the ASR approach based on the ERT method will be given.

Chapter 3 covers the description of the ERT setup. First part deals with the detailed explanation of the ERT setup. Here, the measurement procedure and the components, as well as the algorithms for the automation system will be explained. After that, the limitations of the ERT method with respect to dynamic range and wavefront sensitivity will be given. At the end of this chapter, a detailed error analysis of the ERT method will be discussed using the generalized law of propagation of uncertainty and Monte-Carlo simulations.

In Chapter 4, the results of the experiments that are performed throughout this work will be given. First, the focal length measurements using the ERT setup will be presented. Then, the realized Mach-Zehnder interferometer and SHS setups will be introduced and the results of wavefront aberration measurements will be compared with those of the ERT setup. At the end of this chapter, the results of the aspherical surface measurements with four different aspherical lenses will be given. The results will be compared with the results from a commercial surface profiler.

A summary of the thesis is given in Chapter 5. In addition, suggestions for possible future developments will be described.
2. Theory of experimental ray tracing and aspherical surface retrieval

In the design process of optical systems, optical properties of lenses are usually determined by ray tracing. This is a numerical method which allows calculating direction, position and optical path length of a ray while passing through an optical system. Optical designers use ray tracing software in order to design optical components for intended use. This chapter of the work will begin with the review of the numerical ray tracing method used in this work.

2.1. Definition of a ray

According to Spencer [49], a ray is specified by the coordinates \((x_0, y_0, z_0)\) of a point \(P_0\) and its direction cosines \((X_0, Y_0, Z_0)\) in a reference coordinate system which has its origin at a point \(O\). Fig. 2.1 illustrates the general consideration.

![Ray Diagram](image)

**Figure 2.1.** A ray is defined by its coordinates of origin \((x_0, y_0, z_0)\) and the direction cosines \((X_0, Y_0, Z_0)\).
In general, direction of a ray can be defined by unit vector

\[ \hat{r} = X\hat{i} + Y\hat{j} + Z\hat{k} \]  

(2.1)

where \((\hat{i}, \hat{j}, \hat{k})\) are the unit vectors of the coordinate axes. Here, direction cosines are the projections of unit ray vector \(\hat{r}\) on the coordinate axes \((x, y, z)\)

\[ X = \cos \theta_x, \quad Y = \cos \theta_y, \quad Z = \cos \theta_z \]  

(2.2)

From the geometry, the relationship between the ray slopes along the propagation direction and the direction cosines are simply

\[ T_x = \tan \alpha_x = \frac{X}{Z}, \quad T_y = \tan \alpha_y = \frac{Y}{Z} \]  

(2.3)

The parametric equations for the coordinates of a point along the ray in any given space are given by

\[ x = x_0 + AX, \quad y = y_0 + AY, \quad z = z_0 + AZ \]  

(2.4)

where \(A\) is the optical path length that the ray travels from coordinate start \((x_0, y_0, z_0)\).

### 2.2. Numerical ray tracing

There are various ray tracing methods available for optical system designers. In this work, the general or the skew ray tracing method will be described in detail. An application of this method to an aspherical lens will be presented. The considerations are based on the references \([50,51]\) and the equations are slightly modified.

A complete ray tracing though an optical system consists of four main steps. These are Opening, Transfer, Refraction and Closing. At the first step, starting position and the direction cosines of the ray with respect to the reference surface are determined. Next, ray is transferred to the next surface by calculating the point of intersection of the ray with the next surface. As a third step refraction of the ray at the surface is calculated and the direction cosines of the rays leaving the surface are found. Second and third steps are repeated until the ray is traced through all of the surfaces in the optical system. Finally, ray trace is closed by calculating the path and intersection point of the ray at the last surface.

As mentioned in Chap. 1, an aspherical lens typically consists of an aspherical front surface and a spherical back surface. Fig. 2.2 illustrates ray tracing process through a typical aspherical lens. For simplicity the figure is plotted with two dimensions, using \(y\) and \(z\) axis. However, the following equations are presented including the \(x\) coordinates.
Figure 2.2. Ray tracing through an aspherical lens. A ray is traced from a point \((x_0, y_0, z_0)\) at the reference plane \(\Sigma_0\) to a point \((x_3, y_3, z_3)\) at the observation plane \(\Sigma_3\). A typical aspherical lens with an aspherical front surface \(\Sigma_1\) and a spherical back surface \(\Sigma_2\) is considered.

The main task is to trace a ray from the coordinates \((x_0, y_0, z_0)\) at the reference plane \(\Sigma_0\), passing through the surfaces of the aspherical lens \(\Sigma_1\) and \(\Sigma_2\), to the coordinates \((x_3, y_3, z_3)\) at the observation plane \(\Sigma_3\). It is assumed that there is no change in the orientation of local coordinate system with respect to the reference coordinate. As mentioned above, the process will be handled in four steps. It will start with the Opening step at the reference plane where the position of the ray is \((x_0, y_0, z_0)\) with direction cosines of \((X_0, Y_0, Z_0)\). Note that here a right-handed coordinate system is used and propagation direction of the ray is positive from left to right. Opening is calculated by satisfying the conditions given by

\[
\begin{align*}
    c_0(x_0^2 + y_0^2 + z_0^2) - 2z_0 &= 0 \quad (2.5a) \\
    X_0^2 + Y_0^2 + Z_0^2 &= 1 \quad (2.5b)
\end{align*}
\]

Here, \(c_0\) is the curvature of the reference surface which is taken as zero for a plane surface. This step assures that the ray origin point is located at the reference surface and vector along the ray is a unit vector. Now, the ray has to be transferred to the aspherical front surface. Here, the problem arises from asphericity of the surface. Unlike the spherical surfaces, the local curvature of the asphere changes across its surface. In this work, in order to calculate intersection coordinates of the ray with the aspherical surface, an iterative approximation method is followed. Fig. 2.3 depicts a detailed view of the problem.
As a first approximation, the rays are transferred to the basic spherical part of the aspherical surface. The radius of curvature of this first approximation sphere is taken as the radius of curvature value used in the conventional aspherical surface equation given Eq. 1.1. Transferring of the ray to the 1st approximation sphere begins with computing the \( z \) component of the normal distance between the ray coordinates \((x_0, y_0, z_0)\) and the vertex of the 1st approximation sphere by

\[
e = t_0 Z_0 - (x_0 X_0 + y_0 Y_0 + z_0 Z_0)
\]

\[M_{0z} = (z_0^\prime + e Z_0 + t_0)
\]

where \( t_0 \) is the distance between \( \Sigma_0 \) and \( \Sigma_1 \). Then the normal distance between the ray and the vertex of the 1st approximation sphere is given by

\[
M_0^2 = x_0^2 + y_0^2 + z_0^2 - e^2 + t_0^2 - 2t_0 z_0
\]

and the angle of incidence at the transferred surface is found from

\[
E = \sqrt{Z_0^2 - c_1 (c_1 M_0^2 - 2M_{0z})}
\]

and thereby

\[
L = e + \frac{(c_1 M_0^2 - 2M_{0z})}{Z_0 + E}
\]
where \( L \) is the distance along the ray from the reference plan \( \Sigma_0 \) to the 1st approximation sphere. Then the ray intersection coordinates \((x'_1, y'_1, z'_1)\) at the first approximation sphere can be obtained by computing

\[
\begin{align*}
  z'_1 &= z_0 + LZ_0 - t_0 \\
  y'_1 &= y_0 + LY_0 \\
  x'_1 &= x_0 + LX_0
\end{align*}
\]

(2.10a) \hspace{1cm} (2.10b) \hspace{1cm} (2.10c)

Next step is the iteration process in which series of approximations are done. For this purpose, first, the coordinates \((x'_1, y'_1)\) are substituted into Eq.s 1.1 and 1.2 and the coordinate \(z^*_1\) is found (See Fig. 2.3). The aim is to minimize the difference between \(z'_1\) and \(z^*_1\) at each iteration. For determination of the improved ray intersection coordinates of the ray with the aspherical surface, initially, partial derivatives of the aspherical equation are calculated by

\[
\begin{align*}
  l &= \sqrt{1-c_1r(1+k)} \quad (2.11a) \\
  m &= -y'_1(c_1 + l + \sum_{i=2}^{l} A_{2i}r^{2i-2}) \quad (2.11b) \\
  n &= -x'_1(c_1 + l + \sum_{i=2}^{l} A_{2i}r^{2i-2}) \quad (2.11c)
\end{align*}
\]

and the distance between the intersection coordinates of the ray at the 1st approximation sphere and the actual aspherical surface is found by

\[
G = \frac{l(z^*_1 - z_1)}{(X'n + Y'm + Z'l)}
\]

(2.12)

Now, the new approximated intersection coordinates of the ray with aspherical surface \(\Sigma_i\) are

\[
\begin{align*}
  x^*_1 &= GX + x'_1 \\
  y^*_1 &= GY + y'_1 \\
  z^*_1 &= GZ + z'_1
\end{align*}
\]

(2.13a) \hspace{1cm} (2.13b) \hspace{1cm} (2.13c)

The iteration process is repeated until the error is negligible \(z^*_1 \approx z'_1\). After that, \(z^*_1\) is replaced with the \(z_i\), and the ray intersection coordinates at the aspherical surface \((x_i, y_i, z_i)\) are determined. This final step ends the transferring of the ray to the aspherical surface.

Third step is the calculation of the refraction at the aspherical surface. Computing the equations

\[
\begin{align*}
  P^2 &= l^2 + m^2 + n^2 \quad (2.14a) \\
  F &= Z_0l + Y_0m + X_0n \quad (2.14b) \\
  F' &= \sqrt{P^2(1 - \frac{n_0^2}{n_1^2})} \quad (2.14c)
\end{align*}
\]
leads to the direction cosines of the ray \((X_1, Y_1, Z_1)\) leaving from \(\Sigma_1\) as

\[
X_1 = \frac{n_0}{n_1} X_0 + gn \quad (2.15a)
\]

\[
Y_1 = \frac{n_0}{n_1} Y_0 + gm \quad (2.15b)
\]

\[
Z_1 = \frac{n_0}{n_1} Z_0 + gl \quad (2.15c)
\]

Now, the rays will be transferred to the next spherical surface \(\Sigma_2\). Here, the process described by the Eq.s 2.6 to 2.10 is repeated. After computing the ray intersection coordinates \((x_2, y_2, z_2)\) at the spherical surface \(\Sigma_2\), refraction at the spherical surface shall be considered. Refraction at spherical surfaces is slightly different than that at aspherical surfaces. Equations used are given as

\[
E_1' = \sqrt{\left(1 - \frac{n_1^2}{n_2^2}\right)(1 - E_2)} \quad (2.16a)
\]

\[
g = E_1' - \frac{n_1}{n_2} E_2 \quad (2.16b)
\]

and the direction cosines of the ray departing from the surface \(\Sigma_2\) are

\[
X_2 = \frac{n_2}{n_2} X_1 + gc_2x_2 \quad (2.17a)
\]

\[
Y_2 = \frac{n_2}{n_2} Y_1 + gc_2y_2 \quad (2.17b)
\]

\[
Z_2 = \frac{n_2}{n_2} Z_1 + gc_2z_2 + g \quad (2.17c)
\]

After calculating the direction cosines \((X_2, Y_2, Z_2)\), again using transfer equations in Eq.s 2.6 to 2.10, ray intersection coordinates \((x_3, y_3, z_3)\) at the observation plane \(\Sigma_3\) can be determined. This closes the ray tracing process through an aspherical lens. Eq.s 2.5 to 2.17 are the only necessary equations for ray tracing through aspherical and spherical surfaces. It is possible to apply these equations to any optical system that consists of only spherical and aspherical surfaces.

It should be mentioned that the ray tracing algorithms frequently use the local coordinate systems of individual surfaces. Thus, if there is a change in the orientation of the reference coordinate system and the local coordinate systems at the associated surface, coordinate
transformations should be computed. These calculations are especially useful for introducing tip and tilt of the components in optical systems.

Assume that the local coordinate system \((\bar{x}, \bar{y}, \bar{z})\) is specified with respect to the reference coordinate system \((x, y, z)\) in terms of the Euler angels \((\varphi, \theta, \psi)\) (See Fig. 2.4). The point \(P_0\) lies at the coordinates \((x_0, y_0, z_0)\) of the reference system, direction cosines of the ray are denoted by \((X_0, Y_0, Z_0)\). Here, before computing intersection point of the ray at the next surface, the coordinates of \(P_0\) and direction cosines of the ray should be described in terms of local coordinate system \((\bar{x}, \bar{y}, \bar{z})\).

\[ P_0 = (x_0, y_0, z_0) \]

The relationship between the reference and local coordinate system is defined by the rotation matrix

\[
R = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \theta & \sin \theta \\
0 & -\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
\cos \varphi & 0 & \sin \varphi \\
0 & 1 & 0 \\
-\sin \varphi & 0 & \cos \varphi
\end{bmatrix}
\begin{bmatrix}
\cos \psi & \sin \psi & 0 \\
0 & 1 & 0 \\
-\sin \psi & 0 & \cos \psi
\end{bmatrix}
\]

which describes three successive rotations about \(x, y\) and \(z\) axes. Applying Eq. 2.18 into the ray data yields [49]

\[
\begin{bmatrix}
\bar{x}_0 \\
\bar{y}_0 \\
\bar{z}_0
\end{bmatrix} = R
\begin{bmatrix}
x_0 - \rho_x \\
y_0 - \rho_y \\
z_0 - \rho_z
\end{bmatrix}
\quad \begin{bmatrix}
\bar{X}_0 \\
\bar{Y}_0 \\
\bar{Z}_0
\end{bmatrix} = R
\begin{bmatrix}
X_0 \\
Y_0 \\
Z_0
\end{bmatrix}
\]

(2.19)

where \((\bar{x}_0, \bar{y}_0, \bar{z}_0)\) and \((\bar{X}_0, \bar{Y}_0, \bar{Z}_0)\) are the coordinates of \(P_0\) and the direction cosines of the ray at the local coordinate system, consecutively. \((\rho_x, \rho_y, \rho_z)\) are the offset coordinates between \(O\) and \(\bar{O}\) in the reference coordinate system. After this transformation, ray tracing
process continues with the transfer and the refraction steps. At each case that the following surface has a different orientation than the surface where the ray is traced from, transformation equations should be computed. Otherwise, this step is omitted.

**2.3. Wave and ray aberrations**

In geometrical optics, an ideal optical imaging system transforms a concentric bundle of rays from a point on the object plane to a point on the image plane. Consider an on-axis optical system depicted in Fig. 2.5 [52]. The ideal optical system is represented as a box which can be interpreted as a single or multiple component system. Entrance pupil of the optical system is illuminated by a point source $P$ which produces a spherical input wavefront. Input wavefront will be transformed by the optical system resulting to a spherical output wavefront centered at $P'$. For an ideal optical system, all the rays traced through optical system from object $P$ to image $P'$ have equal optical path lengths. Optical path length (OPL) of a ray is given as

$$OPL = l \cdot n$$  \hspace{1cm} (2.20)

where $l$ is the geometrical path length of the ray and $n$ is the refractive index of the medium. Optical path length of an arbitrary ray $i$ in an ideal optical system can be written as

$$OPL_{\text{ray},i} = OPL_{\text{chief ray}}$$  \hspace{1cm} (2.21)

The condition given in Eq. 2.21 indicates that output wavefront located at the interception point of the exit pupil is spherical. This requires all the rays to pass over $P'$ at the image plane. This image plane is also called as Gaussian or paraxial image plane. $OPL$ can be calculated by using the ray tracing equations given in Sec. 2.2.

Figure 2.5. Illustration of an ideal optical system.
An imperfect optical system does not redirect all the rays from object point to the image point. Such an optical system is called to have “aberrations”. Since there is no point image, the output wavefront can not be a spherical wavefront. When a perfect spherical wavefront with its vertex tangent to the exit pupil is assumed to be the reference wavefront, the deviation between the reference wavefront and the real wavefront is called as “wavefront aberration function”. Radius of the reference wavefront is taken as distance to the paraxial image plane. The reference wavefront is called as Gaussian reference sphere. Wavefront aberration is the optical path difference (OPD) between Gaussian reference sphere and actual wavefront as a function of the exit pupil coordinates \((x, y)\), which is given in wavelength units as

\[
W(x, y) = \frac{\text{OPD}(x, y)}{\lambda}
\]  

(2.22)

In aberrated optical systems, rays emerging from the object point intercept with the image plane at different transverse locations. This phenomenon is illustrated in Fig. 2.6 [53]. Consider a Gaussian reference sphere located with its vertex \(O=(0,0,0)\) at the center of the exit pupil plane. Radius of the Gaussian reference sphere is \(R\) and the center of the reference sphere coincides with paraxial image point \(P'=(x_c, y_c, z_c)\). Let \(W_a\) be the aberrated wavefront at the exit pupil and \(Q=(x,y,z)\) is a point on the aberrated wavefront. The ray passing through the point \(Q\), which is normal to the aberrated wavefront \(W_a\), intercepts the paraxial image plane at \(P''=(x_i, y_i, z_i)\). The transverse distance between points \(P'\) and \(P''\) at the image plane is defined as transverse ray aberrations. These aberrations can be defined by the coordinate pair

\[
\begin{align*}
\varepsilon_x &= x_i - x_c \\
\varepsilon_y &= y_i - y_c
\end{align*}
\]  

(2.23)

There is a direct relationship between the wave aberrations and the ray aberrations. Two alternative derivations of this relationship are reported. Nijboer [54] derived the relationship based on analytical geometry using the coordinate system on the Gaussian reference sphere at the exit pupil. Rayces [55] reported the same relationship by using coordinate system on the aberrated wavefront.
Figure 2.6. Relationship between the wave and the transverse ray aberrations.

According to Rayces’ derivation, wavefront aberration function can be written as the optical path difference between the actual wavefront and the Gaussian reference sphere

\[ W(x, y) = [QP'] - [AP'] \quad (2.24) \]

where \([AP']\) denotes the optical path length between the points \(A\) and \(P'\) and it is equal to the radius of the Gaussian reference sphere \(R\). Relationship between ray and wave aberrations are given as

\[ \frac{\partial W}{\partial x} = -n \frac{\varepsilon_x}{R - W}, \quad \frac{\partial W}{\partial y} = -n \frac{\varepsilon_y}{R - W} \quad (2.25) \]

Since in optical systems \(W\) is in the order of a few wavelengths, it can be assumed that \(W \ll R\). Taking the refractive index of the medium \(n\) as 1, Eq. 2.25 yields to

\[ \frac{\partial W}{\partial x} = -\frac{\varepsilon_x}{R} = \gamma_x, \quad \frac{\partial W}{\partial y} = -\frac{\varepsilon_y}{R} = \gamma_y \quad (2.26) \]

where \(\gamma_x\) and \(\gamma_y\) are the angular aberrations in \(x\) and \(y\) direction. This approximation shows that ray aberrations are directly related to the derivatives of the wave aberrations. Wolf [56] investigated the error term of the approximation in this derivation and found that it’s in 7th order. For practical purposes, very high accuracy can be obtained with this approximation [57].
2.4. Determination of wave and ray aberrations by ray slope measurements

In order to determine wave and ray aberrations in the ERT method, the rays should be traced to the paraxial image plane. Fig. 2.7 illustrates the schematic diagram of the principle. Here, $S_{i(x,y)}^1$ and $S_{i(x,y)}^k$ denote the intersection coordinates of the ray at the first and the $k$th $z$ plane respectively. $S_{i(x,y)}^p$ is the intersection coordinates of the ray at the paraxial image plane.

![Figure 2.7](image)

**Figure 2.7.** Determination of wave and ray aberrations using ERT. Rays are traced to the paraxial image plane to determine the transverse aberration. Thereafter, using Eq. 2.26 wave aberrations can be computed.

It should be noted that the exit pupil plane is assumed to be located at the principle plane of the lens under test. If the test rays are backtraced from the paraxial image plane, the intersection points with the input test rays form a virtual surface. This surface is known as the equivalent refracting locus [58]. The paraxial portion of the equivalent refracting locus coincides with the principle plane of the lens. In case that the object is located at infinity, the image distance, which is the distance between the principle plane and the paraxial image plane, equals to the effective focal length of a lens $f_{\text{efl}}$. Thus, it can be assumed that the radius of the Gaussian reference sphere $R$ equals to the $f_{\text{efl}}$.

In order to calculate the transverse ray aberrations, measured ray positions at different $z$ positions should be traced to the paraxial plane. Assume that $L$ denotes the distance between the $k$th $z$ plane and the paraxial image plane, $L$ can be calculated as
\[ L = \lim_{(x,y) \to (0,0)} \left( R \frac{\sqrt{S_x^2 + S_y^2}}{\sqrt{x^2 + y^2}} \right) \]  

(2.27)

Then, inserting the calculated distance \( L \) to the transfer equations given Sec. 2.2, one can determine intersection coordinates of the rays at the paraxial image plane. Finally, the wavefront aberration function given in Eq. 2.26 can be rewritten as

\[ \frac{\partial W}{\partial x} = -\frac{S_p^x}{f_{\text{eff}}} \quad \frac{\partial W}{\partial y} = -\frac{S_p^y}{f_{\text{eff}}} \]  

(2.28)

### 2.5. Representation of wavefront aberrations with Zernike polynomials

Basic properties of wavefront aberrations are usually described by expanding the wavefront function in polynomial form in the aperture coordinates of the exit pupil. For this purpose, Zernike polynomials are widely used for mathematical representations of wavefront aberrations. These polynomials are a complete set of linearly independent polynomials which are orthogonal over a circle of unit radius [59]. Zernike polynomials are given in polar coordinate system as

\[ Z_{n}^{m}(\rho, \theta) = N_{n}^{m} R_{n}^{m}(\rho) \cos(m\theta) \quad \text{for } m \geq 0, \ 0 \leq \rho \leq 1, \ 0 \leq \theta \leq 2\pi \]

\[ = -N_{n}^{m} R_{n}^{m}(\rho) \sin(m\theta) \quad \text{for } m < 0, \ 0 \leq \rho \leq 1, \ 0 \leq \theta \leq 2\pi \]  

(2.29)

where \( \rho \) is the radial coordinate, \( \theta \) is azimuthal component, \( n \) is the degree of the polynomial and \( m \) is the angular dependence parameter. \( N_{n}^{m} \) is the normalization factor and \( R_{n}^{m}(\rho) \) is the radial polynomial which is given as

\[ R_{n}^{m}(\rho) = \sum_{s=0}^{\left\lfloor \frac{n-|m|}{2} \right\rfloor} \frac{(-1)^s (n-s)!}{s! \left[ 0.5(n+|m|) - s \right]! \left[ 0.5(n-|m|) - s \right]!} \rho^{n-2s} \]  

(2.30)

\[ N_{n}^{m} = \frac{2(n+1)}{1+\delta_{m0}} \quad \delta_{m0} = 1 \quad \text{for } m = 0, \quad \delta_{m0} = 0 \quad \text{for } m \neq 0 \]  

(2.31)

Zernike polynomials can be also defined in the normalized Cartesian coordinate system. Furthermore, a single indexing scheme is also useful for simplicity. In this work, the wavefront will be expressed as weighted sum of Zernike polynomials

\[ W(x,y) = \sum_{j=1}^{N} c_{j} Z_{j}(x,y) \]  

(2.32)
where $j$ denotes the index of the Zernike term. $c_j$ represents the coefficients that are associated with a particular optical property or aberration of the lens under test. It should be noted that $x$ and $y$ are the normalized exit pupil coordinates. The first 15 Zernike terms in Cartesian and polar coordinates and their meanings are given in Table 2.1. Zernike polynomials contain the well known primary (Seidel) aberrations which are classified as spherical aberration, coma, astigmatism, field curvature (defocus) and distortion (tilt). A detailed analysis of the relationship between the primary aberrations and Zernike polynomials is given in [60].

Table 2.1. The first 15 Zernike polynomials and their meanings.

<table>
<thead>
<tr>
<th>Term</th>
<th>Cartesian</th>
<th>Polar</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_1$</td>
<td>1</td>
<td>1</td>
<td>Constant term</td>
</tr>
<tr>
<td>$Z_2$</td>
<td>$x$</td>
<td>$\rho \sin \theta$</td>
<td>Tilt x-direction</td>
</tr>
<tr>
<td>$Z_3$</td>
<td>$y$</td>
<td>$\rho \cos \theta$</td>
<td>Tilt y-direction</td>
</tr>
<tr>
<td>$Z_4$</td>
<td>$2xy$</td>
<td>$\rho^2 \sin(2\theta)$</td>
<td>Astigmatism (45°)</td>
</tr>
<tr>
<td>$Z_5$</td>
<td>$-1 + 2y^2 + 2x^2$</td>
<td>$2\rho^2 - 1$</td>
<td>Focus shift</td>
</tr>
<tr>
<td>$Z_6$</td>
<td>$y^2 + x^2$</td>
<td>$\rho^2 \cos(2\theta)$</td>
<td>Astigmatism (90°)</td>
</tr>
<tr>
<td>$Z_7$</td>
<td>$3xy^2 - x^3$</td>
<td>$\rho^3 \sin(3\theta)$</td>
<td>Triangular astigmatism x</td>
</tr>
<tr>
<td>$Z_8$</td>
<td>$-2x + 3xy^2 + 3x^3$</td>
<td>$(3\rho^3 - 2\rho)\sin \theta$</td>
<td>Third order coma x</td>
</tr>
<tr>
<td>$Z_9$</td>
<td>$-2y + 3x^2y + 3y^3$</td>
<td>$(3\rho^3 - 2\rho)\cos \theta$</td>
<td>Third order coma y</td>
</tr>
<tr>
<td>$Z_{10}$</td>
<td>$y^3 - 3x^2y$</td>
<td>$\rho^3 \cos(3\theta)$</td>
<td>Triangular astigmatism y</td>
</tr>
<tr>
<td>$Z_{11}$</td>
<td>$4y^3x - 4x^3y$</td>
<td>$\rho^4 \sin(4\theta)$</td>
<td></td>
</tr>
<tr>
<td>$Z_{12}$</td>
<td>$-6xy + 8y^2x + 8x^3y$</td>
<td>$(4\rho^4 - 3\rho^2)\sin(2\theta)$</td>
<td>Secondary astigmatism</td>
</tr>
<tr>
<td>$Z_{13}$</td>
<td>$1 - 6y^2 + 6x^2 + 6y^4 + 12y^2x^2 + 6x^4$</td>
<td>$6\rho^4 - 6\rho^2 + 1$</td>
<td>3rd order spherical aberration</td>
</tr>
<tr>
<td>$Z_{14}$</td>
<td>$-3y^2 + 3x^2 + 4y^4 - 4y^2x^2 - 4x^4$</td>
<td>$(4\rho^4 - 3\rho^2)\cos(2\theta)$</td>
<td></td>
</tr>
<tr>
<td>$Z_{15}$</td>
<td>$y^4 - 6x^2y^2 + x^4$</td>
<td>$\rho^4 \cos(4\theta)$</td>
<td></td>
</tr>
</tbody>
</table>

Wavefront aberrations are generally quantified by the P-V and RMS wavefront error. P-V error is the absolute value of the maximum deviation of the actual wavefront from the Gaussian reference sphere. It is only calculated from two data points, the maximum and minimum deviation from the Gaussian reference sphere. Therefore, using only P-V error can be misleading for the overall system performance. It is generally more meaningful to specify the wavefront quality using RMS wavefront error since it is the statistical description of the aberration function that is calculated from all of the measured data [61]. RMS wavefront error is defined as

$$RMS = \sqrt{W^2 - \bar{W}^2}$$ (2.33)
where $\overline{W^2}$ is the mean squared wavefront and $\overline{W}$ is the mean wavefront error. Normalization factor in the Eq. 2.31 is used so that the coefficient of a specific Zernike term represents the standard deviation of the corresponding aberration term. Because of the orthogonality of Zernike polynomials, total RMS wavefront aberration can be written as

$$RMS_{total} = \sqrt{\sum_{k=4}^{20} c_j^2}$$  \hspace{1cm} (2.34)

Note that the first three terms of the Zernike polynomials represent the piston and the tilt in $x$ and $y$ direction. These terms do not affect the quality of the optical characteristics of the wavefront. Therefore, these terms are not considered in the calculation of the total RMS wavefront error.

### 2.6. Wavefront reconstruction

Methods used for describing the wavefront aberration functions in terms of Zernike polynomials depend on the measurement system that is used to detect the wavefront. For example, interferometric measurement which directly measures the phase or the height information of wavefronts requires straightforward method for fitting. It is as well similar for surface profilers, if the measured surface is represented in terms of polynomials. For slope measuring systems, in order to obtain the wavefront data, various wavefront reconstruction methods are available. In this section, two basic wavefront reconstruction methods will be presented: zonal and modal.

#### 2.6.1. Zonal reconstruction

In zonal wavefront reconstruction methods, wavefront retrieval is done using numerical integration techniques. Zonal reconstruction provides point by point description of the wavefront at every measurement point which offers a high resolution wavefront description. According to Southwell numerical integration method [62], average slope between two neighbouring measurement points can be given as

$$S_{i-1,j}^x = \frac{d}{2} (T_x(i-1,j) + T_x(i,j))$$  \hspace{1cm} (2.35)

where $d$ is the distance between two measurement points, $T$ is the slope and $S$ denotes the average slope between two neighbouring points. Consider all the four vertical and horizontal neighbouring measurement points of the point $(i,j)$, the wavefront can be written in four different forms as
Therefore, an estimation for the wavefront at the point \((i, j)\) can be calculated by the weighted average of these four measurements \([63]\):

\[
W(i, j) = \frac{\sigma_{i-1,j} W_{i-1,j} + \sigma_{i+1,j} W_{i+1,j} + \sigma_{i,j-1} W_{i,j-1} + \sigma_{i,j+1} W_{i,j+1}}{\sigma_{i-1,j} + \sigma_{i+1,j} + \sigma_{i,j-1} + \sigma_{i,j+1}} + \frac{\sigma_{i-1,j} S^{x}_{i-1,j} + \sigma_{i+1,j} S^{x}_{i+1,j} + \sigma_{i,j-1} S^{y}_{i,j-1} + \sigma_{i,j+1} S^{y}_{i,j+1}}{\sigma_{i-1,j} + \sigma_{i+1,j} + \sigma_{i,j-1} + \sigma_{i,j+1}}
\]  

(2.37)

where \(\sigma\) is the weighting factor. For circular apertures, \(\sigma\) is one for the measurement points inside the aperture and zero for the points outside of the aperture.

### 2.6.2. Modal reconstruction

Modal wavefront reconstruction involves the expanding wavefront function in polynomial form, i.e. modes. In this section modal wavefront reconstruction with Zernike polynomials will be described and the considerations are based on the approach proposed in Reference \([64]\).

Considering the relationship given in Eq 2.32, the partial derivatives of the wavefront can be approximated by the sum of the derivatives of the Zernike polynomials as

\[
\frac{\partial W(x, y)}{\partial x} = \sum_{j=1}^{n} c_{j} \frac{\partial Z_{j}(x, y)}{\partial x}
\]

\[
\frac{\partial W(x, y)}{\partial y} = \sum_{j=1}^{n} c_{j} \frac{\partial Z_{j}(x, y)}{\partial y}
\]

(2.38)

where \(n\) denotes the number of Zernike terms used in the expansion. The relationship between the gradient of the wavefront and the slopes of the test ray was given by Eq. 1.17. Thereby, the variance of the discrete slope data is given by

\[
Q = \sum_{i=1}^{N} \left( T_{x} - \frac{\partial W(x_{i}, y_{i})}{\partial x} \right)^{2} + \sum_{i=1}^{N} \left( T_{y} - \frac{\partial W(x_{i}, y_{i})}{\partial y} \right)^{2}
\]

(2.39)

and inserting Eq. 2.38 into Eq. 2.39 leads to

\[
Q = \sum_{i=1}^{N} \left( T_{x} - \sum_{j=1}^{n} c_{j} \frac{\partial Z_{j}(x_{i}, y_{i})}{\partial x} \right)^{2} + \sum_{i=1}^{N} \left( T_{y} - \sum_{k=1}^{n} c_{j} \frac{\partial Z_{j}(x_{i}, y_{i})}{\partial y} \right)^{2}
\]

(2.40)
In order to determine the Zernike coefficients $c$, the variance $Q$ is minimized following the condition

$$\frac{\partial Q}{\partial c_k} = 0 \quad \forall k \in \{1, 2...n\}$$

(2.41)

Solving for the Eq.s 2.40 and 2.41 leads to a linear system $A = Bc$ with $k$ equations and $k$ unknown values of the coefficient $c$. Then, Zernike coefficients can be calculated for desired number of terms using the pseudoinverse matrix of $B$ by

$$c = B^{-1}A$$

(2.42)

Finally, the wavefront can be reconstructed by inserting the calculated Zernike coefficients into the Eq. 2.32.

### 2.6.3. Orthogonalization of Zernike polynomials

Orthogonality of Zernike polynomials over a circular pupil offers various advantages in representation of wavefront aberrations. These polynomials represent balanced classical aberrations yielding minimum variance [65]. Moreover, number of polynomials used in the expansion does not affect the value of the coefficients of the polynomials. However, there are two situations that the Zernike polynomials are no longer orthogonal. First, they are not orthogonal over non circular (arbitrary) shaped pupils [66-68]. This is not an important problem for aspherical lens testing, since the apertures have generally circular shapes. Second, Zernike polynomials are not orthogonal over discrete points of the measurement data set [69]. Moreover, for slope measuring techniques, measured wavefront slopes are expanded by the gradients of the Zernike polynomials. But, gradients of Zernike polynomials are also not orthogonal.

A straightforward solution is to use zonal wavefront reconstruction method described in Sec. 2.6.1 and to obtain the wavefront height data from slope measurements. Then, the variance of the discrete wavefront data can be written as

$$Q = \sum_{i=1}^{N} \left[ W(x_i, y_i) - \sum_{j=1}^{n} c_j Z_j(x_i, y_i) \right]^2$$

(2.43)

Here, $W$ is the reconstructed wavefront using numerical integration method. Condition for minimum requires

$$\frac{\partial Q}{\partial c_k} = 2 \cdot \sum_{i=1}^{N} \left[ W(x_i, y_i) - \sum_{j=1}^{n} c_j Z_j(x_i, y_i) \right] \cdot Z_k(x_i, y_i) = 0$$

(2.44)
Rearranging the terms, one obtains

$$\sum_{j=1}^{n} c_j \left[ \sum_{i=1}^{N} Z_j(x_i, y_i) \cdot Z_k(x_j, y_j) \right] = \sum_{i=1}^{N} W(x_i, y_i) Z_k(x_i, y_i) \quad (2.45)$$

and the condition for orthogonality of polynomials in discrete case is

$$\sum_{n=1}^{N} Z_j(x_i, y_i) \cdot Z_k(x_j, y_j) = \delta_{jk} F_j \quad (2.46)$$

where $\delta_{jk}$ is Kronecker delta and

$$F_j = \sum_{n=1}^{N} Z_j^2(x_i, y_i) \quad (2.47)$$

Substituting the orthogonality condition into Eq. 2.45, the coefficients $c$ can be calculated as

$$c_k = \frac{\sum_{i=1}^{N} W(x_i, y_i) \cdot Z_k(x_i, y_i)}{\sum_{i=1}^{N} Z_k^2(x_i, y_i)} \quad (2.48)$$

Eq. 2.50 is only valid, if the Zernike polynomials $Z$ are orthogonal. However, as mentioned above, Zernike polynomials do not satisfy this condition in discrete case. Hence, they should be converted into a new set of orthogonal polynomials. For this purpose, Gram-Schmidt orthogonalization method is used. Here, the procedure described by Malacara et.al. [70] is followed. After generating a new set of orthogonal polynomials, one can determine the coefficients of these new polynomials using Eq. 2.50. Furthermore, coefficients of the original Zernike polynomials can be transformed from the coefficients of the new orthogonal set. However, calculated coefficients of the original Zernike polynomials are still not independent from each other.

Another alternative method in using orthogonal polynomials for fitting is to expand wavefront slopes in Zernike polynomials itself, not to its gradients [71,72]. In this method, Zernike polynomials are also orthogonalized by the Gram-Schmidt method. Derivatives of the wavefront function in terms of the orthogonalized Zernike coefficients is given as

$$\frac{\partial W}{\partial x} = \sum_{j=1}^{N} l_j Z_j(x, y) \quad (2.49)$$

Using the orthogonality condition given in Eq. 2.46, coefficients of the orthogonalized Zernike polynomials can be found as
Next step is the transformation of the wavefront derivatives to the wavefront itself. Therefore, the most common representation of polynomials, the Taylor polynomials are used. Wavefront can be described as linear combination of Taylor polynomials $L$ as

$$W = \sum_{p} a_{p} L_{p}(x,y)$$  \hspace{1cm} (2.51)

Notation for Taylor polynomials is named $L$, because $T$ is used for ray slopes. Partial derivatives of the Taylor polynomials can be written as

$$\frac{\partial}{\partial x} \left( \sum_{p} a_{p} L_{p}(x,y) \right) = \sum_{k} l_k Z_k(x,y)$$

$$\frac{\partial}{\partial y} \left( \sum_{p} a_{p} L_{p}(x,y) \right) = \sum_{k} m_k Z_k(x,y)$$  \hspace{1cm} (2.52)

Coefficients of the Taylor polynomials $a$ can be calculated in terms of the coefficients of the Zernike polynomials $m$ and $n$ using the equations given in Reference [67]. So, the last step is the conversion of the coefficients of the Taylor polynomials to the Zernike polynomials terms.

This complete procedure can be simplified by the following steps.

- Orthogonalization of Zernike polynomials for discrete case using Gram-Schmidt orthogonalization
- Calculation of $l$ and $m$ coefficients by fitting ray slope data directly to orthogonal set of Zernike Polynomial (not to the gradients)
- Determination of $a$ coefficients of the Taylor polynomials from the transformation equations
- Conversion of Taylor polynomials to Zernike polynomials

### 2.7. Determination of the effective focal length from Zernike polynomials

Focal length of a lens is a paraxial property. In this thesis, it is assumed that the Gaussian reference sphere can be approximated by a parabola. Then, wavefront function can be expressed with respect to the effective focal length of the lens as [73]
where \( r_0 \) is the radius of the aperture of the lens and \( f \) is the focal length.

Each term of the Zernike polynomials contains the appropriate amount of each lower term to make it orthogonal to each lower order. Therefore, in order to calculate first order wavefront properties, higher order terms in Zernike polynomials should also be considered. Zernike terms that have parabolic (paraxial) terms up to 4\(^{th}\) order polynomials are given as

\[
W_i(x, y) = c_5 \cdot (-1 + 2y^2 + 2x^2) \\
W_{c5}(x, y) = c_{13} \cdot (1 - 6y^2 - 6x^2 + 6y^4 + 12x^2y^2 + 6x^4) \\
W_{c25}(x, y) = c_{25} \cdot (-1 + 12x^2 + 12y^2 - 30x^4 - 60x^2y^2 - 30y^4 + 20x^6 + 60x^4y^2 + 60x^2y^4 + 20y^6)
\] (2.54)

Neglecting the higher order terms in the polynomials, wavefront can be written as

\[
W_i(x, y) = 2c_5 \cdot (y^2 + x^2) - 6c_{13}(y^2 + x^2) + 12c_{25}(y^2 + x^2)
\] (2.55)

Substituting Eq. 2.55 into Eq. 2.53, focal length of a lens in terms of Zernike coefficients yields [60]

\[
f = \frac{r_0^2}{2 \cdot (2 \cdot c_5 - 6 \cdot c_{13} + 12 \cdot c_{25})}
\] (2.56)

The mathematical description of the focal length using Zernike polynomials given in Eq. 2.56 is restricted up to 4\(^{th}\) order polynomials. However, since Zernike polynomials are complete set of polynomials, it is possible to use an infinite set of Zernike polynomials for the calculation of focal length. An important point is to use only the terms that contain the paraxial terms.

### 2.8. Aspherical surface retrieval

Measurements of aspherical surface profile play a critical role in characterization of aspherical lenses. As discussed in Sec. 1.1.1, the desired information is basically the deviation of the real aspherical surface from the design surface and the deviation from the best-fit radius aspherical surface. In general, any deviation of the aspherical surface affects the optical properties of the lens, hence the optical path difference (OPD). However, deviation in OPD is not simply proportional to the deviation in the surface deviation. In this section, a method that is developed to retrieve the aspherical surface of the lens will be presented. This method includes ray tracing through the lens and calculation of the aspherical surface profile from the slopes of the transmitted rays. A similar method has been proposed by Seong et. al. [74,75]
using a Mach-Zehnder interferometer which is called reverse ray tracing. There, rays are traced from the detector back to the test sample including the transformation optics and using the information of the OPD of the rays, surface SAG of the lens is determined. Here, since ERT does not need any transformation optics, only the test sample will be considered, and calculations will be based on the ray slopes and derivatives of the surfaces. Seong et. al. also neglected the required coordinate transformation from the detector plane to the lens. In reference [76], the non-paraxial relationship between phase transmittance function and a single refracting surface is described including the coordinate transformation. The coordinate system of the lens surface is determined by a transformation of the coordinate system of the detector plane where the phase (or OPD) information is gathered. The most important advantage of ERT in surface reconstruction is that the coordinate system on the test lens is already known by the nature of the method.

2.8.1. Aspherical surface retrieval for plane second surface

Consider a plano-convex aspherical lens as shown in the Fig. 2.8. For simplicity, calculations will be performed only in $y$ and $z$ axes. Consider a ray parallel to the optical axis with a transverse coordinate $y$. This ray is incident on the surface $Z_1$ at position $y$ with an angle $\alpha$ with respect to the surface normal. Partial derivative of the Surface $Z_1$ can be defined as

$$\frac{dZ_1}{dy_1} = \tan \alpha \quad (2.57)$$

From Snell’s Law, the refraction at the surface $Z_1$ can be simply written as

$$\sin \alpha n_1 = \sin \alpha' n_2 \quad (2.58)$$

Because of the negative relationship between $\alpha$ and $\beta$, it is clear that

$$\alpha' = \alpha + \beta \quad (2.59)$$

Inserting Equation 2.61 into 2.60, one leads to

$$\sin \alpha n_1 = \sin(\alpha + \beta)n_2$$
$$\sin \alpha n_1 = (\sin \alpha \cos \beta + \sin \beta \cos \alpha)n_2 \quad (2.60)$$

Dividing both sides of the equation by $\cos \alpha$ gives

$$\tan \alpha n_1 = (\tan \alpha \cos \beta + \sin \beta)n_2 \quad (2.61)$$

$\tan \alpha$ can be substituted as

$$\tan \alpha = \frac{-\sin \beta n_2}{\cos \beta n_2 - n_1} \quad (2.62)$$
Therefore, the surface slope can be written in terms of $\beta$ as

$$\frac{dZ_1}{dy_1} = -\frac{\sin \beta n_2}{\cos \beta n_2 - n_1}$$  \hspace{1cm} (2.63)$$

Refraction at the second surface by the Snell’s law is simply given by

$$\sin \beta n_2 = \sin \beta' n_1$$ \hspace{1cm} (2.64)$$

Solving for $\sin \beta$ and inserting in Eq. 2.63 gives

$$\frac{dZ_1}{dy_1} = -\frac{\sin \beta' n_1}{\cos \beta n_2 - n_1}$$ \hspace{1cm} (2.65)$$

In ERT, the measured parameter is $\beta'$ and $\beta$ is unknown. Using the trigonometric relation and the Eq. 2.64, $\beta$ can be written in terms of $\beta'$ as

$$\cos \beta = \sqrt{1 - \left( \frac{\sin \beta' n_1}{n_2} \right)^2}$$ \hspace{1cm} (2.66)$$

Therefore, Eq. 2.65 can be rewritten as

$$\frac{dZ_1}{dy_1} = -\frac{\sin \beta' n_1}{n_2 \sqrt{1 - \left( \frac{\sin \beta' n_1}{n_2} \right)^2} - n_1}$$ \hspace{1cm} (2.67)$$
which shows the relationship between the slope of the first surface and outgoing ray angle. Here, it can be seen that thickness $t$ of the lens has no influence on the retrieved aspherical surface of the lens.

### 2.8.2. Aspherical surface retrieval for non-planar second surface

In case that the second surface has a spherical shape, the incidence angle of the ray at the second surface $Z_2$ is not simply $\beta$ as in the previous section. As illustrated in Fig. 2.9, the incident angle $\theta$ depends on the slope of the second surface at intersection coordinate of the ray at the second surface, $dZ_2/dy_2$. Derivative of the second surface can be defined by the geometry as

$$\frac{dZ_2}{dy_2} = \tan \gamma$$  \hspace{1cm} (2.68)

Using the geometrical relationships $\theta = \gamma - \beta$ and $\theta' = \gamma - \beta'$ ($\beta$ is negative for $\gamma > 0$), refraction at the second surface by the Snell’s Law is given as

$$\sin(\gamma - \beta)n_2 = \sin(\gamma - \beta')n_i$$  \hspace{1cm} (2.69)

Applying the trigonometric equations as in Eq. 2.64 and Eq. 2.66, derivative of the second surface is obtained by

$$\frac{dZ_2}{dy_2} = \tan \gamma = \frac{\sin \beta n_2 - \sin \beta' n_i}{\cos \beta n_2 - \cos \beta' n_i}$$  \hspace{1cm} (2.70)

Then, $\sin \beta$ is substituted as

$$\sin \beta = \left[ \sin \beta' n_i + \frac{dZ_2}{dy_2} (\cos \beta n_2 - \cos \beta' n_i) \right] \cdot 1/n_2$$  \hspace{1cm} (2.71)

Inserting Eq. 2.71 into Eq. 2.63, the derivative of the first surface of the lens is found as

$$\frac{dZ_1}{dy_1} = \frac{\sin \beta' n_i - \frac{dZ_2}{dy_2} (\cos \beta n_2 - \cos \beta' n_i)}{\cos \beta n_2 - n_i}$$  \hspace{1cm} (2.72)
Eq. 2.73 indicates that the derivative of the first surface depends on the angle $\beta'$ of the transmitted ray, the second surface $Z_2$, intersection coordinate of the ray at the second surface $y_2$, the angle $\beta$ and refractive indices $n_2$ and $n_1$. It is also obvious that the $y_2$ is dependent on the thickness $t$ of the lens. Here, rest of the considerations will be presented with the assumption that $Z_2$, $t$ and the refractive index of the lens $n_2$ are known. Moreover, refractive index of air $n_1$ will be taken as 1.

As already mentioned, the ERT measures the angle $\beta'$ of the transmitted ray for the given input coordinates $y$. For the calculation of the aspherical surface of the lens by Eq. 2.72, the unknown parameters are the angle $\beta$ and $y_2$. These parameters can not be directly measured with ERT. To determine these parameters, an optimization process is proposed.

Suppose that the complete aspherical lens is modelled numerically and $Z_1^M$ is the modelled first surface. The mathematical description of $Z_1^M$ is the aspherical surface equation and the parameters of the aspherical surface equation, which are the radius of curvature $R$, conic constant $k$ and the aspherical coefficients $A$, are variable. The use of aspherical equation is not required; different polynomial series can be used. However, aspherical surface equation is customary used in manufacturing and metrology of aspherical lenses and the numerical ray tracing routine implemented in this work uses the aspherical surface equation.

The relationship between the model first surface and the actual first surface is assumed to be
Then, according to Eq. 2.72, the model first surface can be written as

\[
\frac{d(Z_i^M)}{dy_i} = \frac{d(Z_i - \Delta)}{dy_i} = \frac{\sin \beta^M - \frac{dZ_2}{dy_2^M} (\cos \beta^M n_2 - \cos \beta^M) + \sin \beta^M}{\cos \beta^M n_2 - 1}
\]

(2.74)

where \( M \) denotes that the variable belongs to the model function. Note that, since \( Z_2 \) is known, it’s not a variable in the model function. It is a clear fact that further the model \( Z_i^M \) gets closer to the actual \( Z_1 \), \( \cos \beta^M \) gets closer to the actual \( \cos \beta \), then \( y_2 \) to \( y_2^M \) and \( \beta^M \) to actual \( \beta' \). The condition can be expressed as

\[
\lim_{(z_i, z_i') \to 0} (\cos \beta - \cos \beta^M, y_2 - y_2^M, \sin \beta' - \sin \beta'^M) = 0
\]

(2.75)

Hence, in case that \( Z_i^M = Z_i \), which means perfect description of the actual surface \( Z_1 \) with the model surface \( Z_i^M \), the angle of the transmitted ray of the model \( \beta^M \) and the actual lens \( \beta' \) are identical. In order to achieve this condition, a least squares minimization process can be performed by

\[
Q = \sum (Z_i - Z_i^M (R, k, A_2, A_4, ..., A_n))^2 = \sum (\Delta)^2
\]

(2.76)

where \( R, k \) and \( A \) are the variable parameters in the optimization process. Naturally, there might be some conditions where \( Z_i^M \) does not perfectly represent \( Z_i \). Then, for very small values of \( \Delta \), it may be assumed that \( \cos \beta^M \approx \cos \beta \). Using this assumption the only unknown parameter \( \cos \beta \) in Eq. 2.72 can be substituted with \( \cos \beta^M \). Thereby, subtracting Eq. 2.74 from Eq. 2.72, the difference between the actual and the model surface can be found by

\[
\frac{d(Z_1 - Z_i^M)}{dy_i} = \frac{d\Delta}{dy_i} = \frac{\sin \beta' - \sin \beta^M + (\frac{dZ_2}{dy_2})^M [\cos \beta' - \cos \beta^M]}{\cos \beta^M n_2 - 1}
\]

(2.77)

Finally, the actual surface \( Z_1 \) can be calculated simply by the summation of \( \Delta \) and \( Z_i^M \) as indicated in Eq. 2.73. The assumption \( \cos \beta^M \approx \cos \beta \) is not valid for large \( \Delta \) values. This also means that Eq. 2.77 is not valid. Thus, the validity of the of the approximation strongly depends on the degree of the representation of the \( Z_i \) by the model surface \( Z_i^M \). The limitations will be presented in the next section using simulations.

In order to generalize the idea of ASR by transmitted rays, it is useful to define the process by operators. Mathematically, the operation of refraction at an arbitrary surface can be defined as
Theory of experimental ray tracing and aspherical surface retrieval

\[ r' = R[r; Z(y)] \]  

(2.78)

where \( r \) is the incident ray vector on the arbitrary surface \( Z \), \( R \) is the refraction operation and \( r' \) is the refracted ray. The ray vector \( r \) is defined by its coordinates and the direction cosines (Eq. 2.1). Using the operator notation, refraction processes at two successive surfaces of the lens can be expressed as

\[ r' = R[r; Z_1(y_1)] \]

(2.79)

\[ r'' = R[r'; Z_2(y_2)] \]

(2.80)

where \( r'' \) is the transmitted ray, \( y_1 \) and \( y_2 \) are local intersection coordinate of the ray with the surfaces \( Z_1 \) and \( Z_2 \). Inserting Eq.2.79 into 2.80 leads to

\[ r'' = R \left[R[r; Z_1(y_1)]; Z_2(y_2)\right] \]

(2.81)

which describes both of the refraction processes on the lens surfaces. It can simply be seen that outgoing ray vector \( r'' \) is dependent on the parameters of the refracted ray from the first surface \( Z_1 \) and its intersection coordinate \( y_2 \) at the surface \( Z_2 \). ERT method delivers the information about the outgoing ray \( r'' \) and the primary assumption is that the surface \( Z_2 \) is known. In this situation, only the aspherical surface \( Z_1 \) remains unknown. Modelling of the complete process gives

\[ r''^M = R \left[r'^M; Z_2(y_2^M)\right] = R \left[R[r; Z_1^M(y_1)]; Z_2(y_2^M)\right] \]

(2.82)

where \( r''^M \) is the modelled outgoing ray using modelled aspherical surface \( Z_1^M \). Then, for \( Z_1^M = Z_1 \), \( r''^M \) would be equal to \( r' \) and thereby \( y_2^M = y_2 \).

### 2.8.3. Simulations of the aspherical surface retrieval

In order to verify the ASR approach described in the previous section, simulations are performed. The idea is to generate certain aspherical surface profiles including certain high frequency surface deformations and to retrieve them by the ASR. As a first step, an aspherical lens with given parameters is simulated by the commercial optical design program Zemax. With this software virtual rays are traced through the simulated lens and the angles of the transmitted rays are determined. Using a commercial software also provides a comparison to the numerical ray tracing algorithms that are written during this work.

The required parameters to simulate an aspherical lens are the geometry of the first surface \( Z_1 \), the second surface of the lens \( Z_2 \), the glass type (for the refractive index) and the thickness \( t \) of the lens. \( Z_1 \) is considered to be the aspherical surface and it is described by the equation
where $Z_i^s$ is the simulated aspherical surface of the lens which is the sum of the base aspherical surface $Z_i^b$ and the residual surface deformations $\Delta^s$. Here, $Z_i^b$ can be considered as the design aspherical surface $Z_d$ and $Z_i^a$ as the actual surface profile of the lens $Z_a$. So, it is a similar relationship which is defined by Eq. 1.3. The base surface $Z_i^b$ is simulated by the aspherical surface equation including the radius of curvature $R$, the conic constant $k$ and the aspherical coefficients $A_{2i}$. The parameters of the base aspherical surface that are used in the simulations are given in Table 2.2.

### Table 2.2. Aspherical surface parameters of the lens used in the simulations.

<table>
<thead>
<tr>
<th>$R_i$</th>
<th>$k$</th>
<th>$A_2$</th>
<th>$A_4$</th>
<th>$A_6$</th>
<th>$A_8$</th>
<th>$A_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>32.5</td>
<td>-1.71664</td>
<td>7.0552e-04</td>
<td>4.10114e-06</td>
<td>-9.42e-10</td>
<td>1.32564e-13</td>
<td>-6.4285e-17</td>
</tr>
</tbody>
</table>

Residual surface deformations $\Delta^s$ are considered to periodic oscillations with a given frequency and amplitude. They are generated by the equation

$$\Delta^s(y) = a \cdot \cos \left( \frac{2\pi i}{d} \cdot y \right)$$

where $a$ is the amplitude, $d$ is the diameter of the lens. $i/d$ defines the spatial frequency $f$ of the oscillation.

The procedure of the simulation process can be explained by the following steps:

- Generation of $Z_i^s$ with the given parameters of $Z_i^b$ and $\Delta^s$ and import to Zemax
- Input parameters $n, t, Z_2$ to Zemax
- Ray tracing in Zemax and export slopes $T_y$ of the transmitted rays and test ray positions $y$
- Import $T_y$ and $y$ into the ASR routine and calculation of aspherical surface profile $Z_i^c$
- Comparison of the simulated aspherical surface profile $Z_i^s$ with the calculated aspherical surface profile $Z_i^c$
- Calculation of the standard deviation $\sigma$ of $(Z_i^s - Z_i^c)$

Calculated standard deviation at the last step of the simulation procedure is considered to be the quantitative description of the ASR approach. The simulations are performed at different
spatial frequencies \( f \) of the periodic oscillations. The base aspherical surface profile \( Z_i^b \) is kept the same at each simulation. Spatial frequencies are varied from 0.05 mm\(^{-1}\) to 0.5 mm\(^{-1}\) with steps of 0.05 mm\(^{-1}\). At each situation, the above defined simulation process is repeated. The amplitude \( a \) of the periodic oscillations is taken 0.5 \( \mu \)m.

2.8.3.1. Simulations for plane second surface

A plano-convex aspherical lens is simulated with a perfect second plane surface (\( R_2 = \infty \)) and the thickness of the lens \( t \) is 12.5 mm. The aspherical surface \( Z_i^p \) is retrieved using the Eq. 2.67. The simulated 3D periodic oscillations \( \Delta^s \) for \( f = 0.05 \) mm\(^{-1}\) and \( f = 0.25 \) mm\(^{-1}\) are illustrated in Fig. 2.10a and 2.10b. For these cases, the calculated periodic oscillations \( \Delta^c \) by the ASR approach are shown in Fig. 2.10c and 2.10d. Extreme correlation between \( \Delta^s \) and \( \Delta^c \) can clearly be seen. Furthermore, cross-sections of \( \Delta^c \) along \( y \) axis are plotted in Fig. 2.10e and 2.10f with the blue line. In the same figures, the deviations \( \Delta^s - \Delta^c \) are plotted with the red line. For \( f = 0.05 \) 1/mm in Fig. 2.10e, \( \sigma(Z_i^s - Z_i^c) \) is calculated as 0.24 nm. On the other side, \( \sigma(Z_i^s - Z_i^c) \) for \( f = 0.25 \) mm\(^{-1}\) is found as 0.55 nm. In both cases, the deviations are even below nanometer region.

The calculated standard deviations \( \sigma(Z_i^s - Z_i^c) \) with respect to spatial frequency of oscillations \( f \) are shown in Fig. 2.11. Here, it can be seen that \( \sigma(Z_i^s - Z_i^c) \) increases when \( f \) is increased. This was an expected result since the number of rays traced through the simulated lens is kept the same in all simulations. Therefore, increasing \( f \) leads to higher sampling errors in the simulations. This effect can also be seen in Fig. 2.10f. However, \( \sigma(Z_i^s - Z_i^c) \) is still in the order of several nanometers. Moreover, this error can be reduced by simply increasing the number of traced rays. These results verify the ASR approach for planar second surface.
Figure 2.10. Simulations of an aspherical lens with a plane second surface a) $\Delta^s$ for $f = 0.05$ mm$^{-1}$ b) $\Delta^s$ for $f = 0.5$ mm$^{-1}$ c) $\Delta^c$ for $f = 0.05$ mm$^{-1}$ d) $\Delta^c$ for $f = 0.25$ mm$^{-1}$ e) Cross-section of $\Delta^c$ and $(Z^s - Z^c)$ for $f = 0.05$ mm$^{-1}$ f) Cross-section of $\Delta^c$ and $(Z^s - Z^c)$ for $f = 0.25$ mm$^{-1}$. 
Figure 2.11. Standard deviation of the difference between simulated and reconstructed aspherical surface $\sigma \left( Z_i^r - Z_i^c \right)$ with respect to spatial frequency of oscillations $f$.

2.8.3.2. Simulations for non-planar second surface

For the simulations of an aspherical lens with a non-planar second surface, $Z_2$ is considered to have a perfect spherical shape with $R_2 = 65$ mm. The same methodology is applied as in the previous section and identical simulated aspherical surfaces $Z_i^r$ are imported to the ASR routine. This time, analysis is done with the method given for aspherical lenses with non-planar second surfaces. In the optimization process, variable parameters of the model aspherical surface $Z_i^M$ are $R$, $k$, and $A$ up to 20th order.

The retrieved cross-sections of $\Delta^c$ for spatial frequencies $0.05$ mm$^{-1}$, $0.25$ mm$^{-1}$ and $0.35$ mm$^{-1}$ are illustrated with the blue line in Fig. 2.12a, 2.12b and 2.12c, respectively. The deviations $Z_i^r - Z_i^c$ are plotted again with the red line for each case. A good agreement between $Z_i^r$ and $Z_i^c$ for the cases $f = 0.05$ mm$^{-1}$ and $f = 0.25$ mm$^{-1}$ can be seen. The standard deviations $\sigma \left( Z_i^r - Z_i^c \right)$ are 0.21 nm and 0.24 nm, respectively. However, for $f = 0.35$ mm$^{-1}$, $\sigma \left( Z_i^r - Z_i^c \right)$ is significantly increased to 16.7 nm. This can be attributed to the fact that the model aspherical surface $Z^M$ does not exactly represent the actual aspherical surface at this frequency of oscillations. In order to investigate this situation; the same data set is analyzed in with variable parameters up to 10th order aspherical surface terms. For simplification, two cases are defined as
• Case 1: \( Z_i^M \{R, k, A_2, A_4, \ldots, A_{10}\} \)

• Case 2: \( Z_i^M \{R, k, A_2, A_4, \ldots, A_{20}\} \)

Here, Case 2 is expected to represent higher frequency oscillations of the aspherical surface compared to Case 1. The results for both cases are shown in Fig. 2.12, where \( \sigma(Z_i^r - Z_i^c) \) is presented with respect to spatial frequency of oscillations \( f \). For Case 1, it is clear that \( \sigma(Z_i^r - Z_i^c) \) starts to increase abruptly in the frequency region of 0.15 mm\(^{-1}\). Then, for \( f > 0.25 \) mm\(^{-1}\), \( \sigma(Z_i^r - Z_i^c) \) stagnates and stays around approximately 26 nm. It can be stated that for \( 0 < f \leq 0.1 \) mm\(^{-1}\), \( Z_i^M \) perfectly represents the actual surface of \( Z_i^r \) and thus \( \cos \beta^M = \cos \beta \). (Note that at this statement, numerical and sampling errors are neglected).

For \( f > 0.25 \) mm\(^{-1}\), \( Z_i^M \) can not exactly describe the actual surface \( Z_i^r \) anymore. This is the region that the assumptions \( Z_i^M \approx Z_i^r \) and \( \cos \beta^M \approx \cos \beta \) are used. When the Case 2 is observed, \( \sigma(Z_i^r - Z_i^c) \) starts to increase at \( f = 0.3 \) mm\(^{-1}\). Therefore, for \( 0 < f \leq 0.25 \) mm\(^{-1}\), it can be assumed that \( Z_i^M = Z_i^r \) and \( \cos \beta^M = \cos \beta \). In general, conditions for Case 1 can be given as

\[
\begin{align*}
Z_i^r &= Z_i^M \cos \beta = \cos \beta^M & \text{for} & & 0 < f \leq 0.10 \\
Z_i^r &\approx Z_i^M, \cos \beta \approx \cos \beta^M & \text{for} & & f > 0.10
\end{align*}
\] (2.85)

and conditions for Case 2 are

\[
\begin{align*}
Z_i^r &= Z_i^M \cos \beta = \cos \beta^M & \text{for} & & 0 < f \leq 0.25 \\
Z_i^r &\approx Z_i^M, \cos \beta \approx \cos \beta^M; & \text{for} & & f > 0.25
\end{align*}
\] (2.86)

So, higher the order of aspherical terms used in the optimization process, higher the accuracy of the ASR approach. The simulations results are depicted in Fig. 2.13.

It can be concluded that the description of \( Z_i^r \) with the model \( Z_i^M \) has a critical role in the ASR approach. The optimum condition is \( Z_i^r = Z_i^M \). For non-rotationally symmetric surface undulations odd terms of the aspherical surface equation shall be considered. However, there is a practical limitation in the number of the aspherical terms used in the minimization process. As a consequence, it is impossible to obtain the condition \( Z_i^r = Z_i^M \). Thus, the assumption which results to Eq. 2.77 has to be used. In the interval that the assumption is used, the calculated standard deviations \( \sigma(Z_i^r - Z_i^c) \) are approximately 26 nm which is % 2.5 of the P-V of the simulated periodic surface oscillations \( \Delta^r \).
Figure 2.12. Simulations of an aspherical lens with a plane second surface a) Cross-section of $\Delta^c$ and $(Z'_i - Z^c_i)$ for $f = 0.05$ mm$^{-1}$ b) Cross-section of $\Delta^c$ and $(Z'_i - Z_i)$ for $f = 0.25$ mm$^{-1}$ c) Cross-section of $\Delta^c$ and $(Z'_i - Z_i)$ for $f = 0.35$ mm$^{-1}$.
Figure 2.13. Standard deviation of the difference between simulated and reconstructed aspherical surface $\sigma(Z_s^r - Z_s^i)$ with respect to spatial frequency of oscillations $f$. In Case 1, up to $10^{th}$ order aspherical terms are used as variables in the model aspherical surface $Z^M$. In Case 2, maximum aspherical term is $20^{th}$ order.
3. Description of experimental ray tracing setup

ERT method is based on the principles of numerical ray tracing. In numerical ray tracing, a virtual ray is traced through surfaces of the optical component or the optical system. By this way, intersection coordinates of the ray with the individual surfaces, direction cosines of the incident and refracted rays can be computed. In ERT method, a narrow test ray is physically traced through the optical component under test. In this work, an ERT setup is realized in a lab environment. In this section, the realized ERT setup will be presented. Furthermore, the properties and limitations of the ERT method will be defined in detail.

3.1. Measurement setup - Hardware

The ERT setup consists of five main parts. These are the light source, XY motorized translation stages, the camera, the Z translation stage and the lens holder. Fig. 3.1 illustrates the schematic diagram of the system and an image of the setup is shown in Fig. 3.2. In this section, each part will be discussed separately.

Figure 3.1. Schematic diagram of the ERT setup.
3.1.1. Light source (test ray)

The selection of the light source is flexible for ERT setup compared to interferometers. A monochromatic and spatial coherent light source is not required, because only the location of the spot (centroid) on the camera is measured. Two alternatives can be taken into consideration: a single wavelength or a broad spectrum light. The selection between these two possibilities depends on the application. For example, if the measurement of an optical property such as focal length with respect to wavelength is required, then a broad spectrum light source can be utilized. A halogen bulb or a LED can be used and selection of test wavelength can be simply done by a bandpass filter. Measurements can be performed using different bandpass filters at different wavelengths. Fig. 3.3 shows the setup that generates a test ray from a halogen bulb.

Light beam from the halogen is focused to the pinhole P1 using the focusing lens L1. In order to produce a highly collimated beam, light beam is spatially filtered using a P1. This beam is collimated using the lens L2. After passing through the bandpass filter for the selection of the test wavelength, a second pinhole P2 is used to create a narrow test ray. Here, alignment of
the lenses and the pinholes are critical in order to avoid loss of energy while light passing through the pinholes.

![Figure 3.3](image1.png)

**Figure 3.3.** Photograph of the setup that generates a test ray from a halogen bulb. L1 focuses the beam to the pinhole P1 which spatially filters the light. L2 is used to collimate the beam and P2 creates a narrow test ray.

The other alternative is to use a laser diode as a light source. In this case, the measurement wavelength is restricted to the emitting wavelength of the laser diode. In this work, a setup is implemented using a laser diode emitting at a center wavelength of 635 nm with a peak power of 2 mW. Laser output light is coupled into a 4 µm single mode fiber. At the output of the fiber, a commercial collimator is mounted. It is an adjustable aspherical collimator producing a beam waist radius of 165 µm. The implemented setup with the laser diode is depicted in Fig. 3.4.

![Figure 3.4](image2.png)

**Figure 3.4.** Test ray produced by a fiber coupled laser diode and an aspherical collimator.
3.1.2. **XY translation stages**

One of the most important components in the ERT setup is the XY translation stages. The complete system producing the test ray is mounted on these two linear stages. They allow the movements of the test ray along $x$ and $y$ axis. In this work, Aerotech 50ANT-LX is utilized. Maximum travel distance of both of these stages is 50 mm which as well restricts the maximum measurable aperture of a test lens to 50 mm. Repeatability of the positioning system in bi-directional movement is 25 nm. Both linear stages are controlled by the Ensemble Stand-Alone Multi-Axis motion controller via Ethernet communication. This controller has an in-built unit which delivers the position feedback. It can be programmed for continuous scanning purposes, sending the required position data back to the user. The linear stages can be driven at a maximum speed of 200 mm/s with a positioning resolution of 1 nm.

3.1.3. **Z translation stage**

The movement of the camera along the propagation direction of the test ray is done using a single Z translation stage. A linear translation stage from OWIS Limes90, DS30 motor with an integrated linear glass scale is implemented. As a controller Owis DC500 is utilized, it is connected to the main computer via RS 232 interface. This complete Z translation allows a resolution of 100 nm and a travel distance of 120 mm.

3.1.4. **Camera (detector)**

A CMOS camera is used as a detector in the ERT setup. This camera has 1312 (Horizontal) by 1024 (Vertical) pixels each having dimensions of 8 µm x 8 µm. The total active is 10.496 mm by 8.192 mm. At full resolution 68 frames per second can be read out. The camera is connected to PC via the GigE interface.

3.1.5. **Lens holder**

The lens holder is utilized in the measurement setup in order to locate the test lens in the measurement. This requires the positioning of the test lens along the optical axis of the measurement system. Furthermore, lens position with respect to the angles around $x$ and $y$ axis should be aligned. In this measurement setup, a commercial 5-axis adjustable Newport lens holder is used. Maximum lens diameter is 50.8 mm and it allows XYZ movement and XY angular adjustments about two orthogonal and coplanar axes.
3.2. Controlling unit (Software)

During this work, controlling software is developed for automating the complete process of the measurement. The program is written in the environment “LabWindows/CVI by National Instruments” and the programming language is C. In addition, a Graphical User Interface (GUI) is developed to control the complete measurement process. Flowchart describing the controlling unit software is given in Appendix A. In this section, complete measurement process will be discussed in detail.

3.2.1. Measurement procedure

Measurement procedure of the ERT setup can be described step by step as follows:

- Initialization of the system components
- System calibration (if necessary)
- Placing the test lens at the lens holder and manual alignment according to the measurement coordinate system
- Definition of the measurement field diameter and spacing between scan positions
- Setting linear stage speed
- Selection of the scan type
- Definition of the number and the locations of the \( z \) positions of the camera
- Start scanning
- Save measurement data to output file
- Analysis of measurement results

3.2.2. Scanning structure

There are two different scanning options available for the ERT setup. First, a linear scan can be done along the \( x \) or \( y \) axis of the coordinate system. This requires manually alignment of the device under test in the measurement system. After finding the center of the lens, a fast linear scan can be done for a given diameter of the test lens and the spacing between scanning positions. Number of valid scan positions for a linear scan is given as

\[
N = \frac{D}{dy} + 1 \quad (3.1)
\]
where $D$ is diameter of the measurement field and $dy$ is the spacing between scan positions. Fig. 3.5 illustrates an example of a linear scan in the measurement coordinate system.

![Figure 3.5](image)

Figure 3.5. Example of a single line scan geometry. After defining the diameter of the test lens and spacing between the scan positions, test lens is aligned to the axis of the measurement system. Scan can be done along y or x axis.

For testing the complete aperture of the test lens, meander scan geometry is used (see Fig. 3.6). Generally, concentric scan can also be used in ERT setup as proposed in reference [48]. For that, rather than using two linear stages for XY translation, combination of a single linear stage with a rotary stage would be more suited. At this alternative, performing linear and polar scans around the lens would provide a 3D measurement.

![Figure 3.6](image)

Figure 3.6. Meander scan geometry. All scan positions inside the lens aperture are defined by the diameter of the measurement field and the number of edge measurement points.
3.2.3. **Measurement speed**

Measurement speed of the ERT setup depends on the scan type that is selected by the user at the beginning of the measurement. Obviously, compared to meander scan, line scan is a faster method. Especially for rotationally symmetric aspherical lenses, line scans across \( x \) and \( y \) axes are generally sufficient to have a good representation of the aspherical surface profile. For determination of the effective focal length, the 3D wavefront aberration function and the 3D aspherical surface profile, it is necessary to use the meander scan.

Two important components in the ERT setup that influence the measurement speed are the XY translation stages and the camera. The controller of the XY translation stages “Ensemble” includes “Position Synchronized Output” (PSO) feature. The PSO is programmed to generate trigger signals which are synchronized to the desired scan positions. The acquisition of the image by the camera is synchronized to the trigger signal. The time required for a single acquisition \( dt_a \) is simply the sum of the exposure time and the read out time. The camera is idle until it gets a new trigger signal from the PSO. After the acquisition is completed, the captured image is processed and the centroid positions are calculated. At the end, the calculated data is written into the measurement data file. So, time duration required in between two successive trigger signals \( dt \) should be longer than the sum of acquisition time \( dt_a \) and the process time \( dt_p \). By introducing a buffer time \( dt_b \) to ensure valid measurements, minimum time duration required in between two trigger signals \( dt_{min} \) can be written as

\[
dt_{min} = dt_a + dt_p + dt_b
\]  

(3.2)

Here, time delays in between each process are neglected. If the trigger signal is fired before the acquisition of one image is completed, the camera simply misses the following image and stays idle until it gets a new trigger signal. Fig. 3.7 illustrates the timing diagram of the complete process.

**Figure 3.7.** Timing diagram of a linear scan.
The maximum velocity of the translation stages that can be used during a measurement is calculated as

\[
v_{\text{max}} = \frac{dy}{dt_{\text{min}}}
\]  

where \(dy\) is distance between two scan positions. Using the number of valid positions in a line scan given in Eq. 3.1, the measurement time of line scan can be calculated by

\[
t_d \approx \frac{D}{v_{\text{max}}} n_z + t_{dz}(n_z - 1)
\]

where \(t_d\) is the measurement time and \(n_z\) is the number of \(z\) positions used during the measurement. \(t_{dz}\) is the time required for the movement in between two given \(z\) positions. Here, the first term of Eq. 3.4 defines the time duration of XY scanning process and the second term is the time required for the movements between \(z\) positions. An example with given measurement parameters and the calculated measurement time is presented in Table 3.1. The parameters \(dt_a\) and \(dt_p\) are measured as 10 ms and 5 ms respectively. The buffer time \(dt_b\) is taken 2 ms. It can be seen that for a linear scan of 40 mm diameter aspherical lens, required measurement time \(t_d\) is found 15.6 seconds.

| Table 3.1. Measurement time required for an example line scan test. |
|-------------------|-----------|
| **Input** | **Output** |
| \(D\) | 40 mm |
| \(dy\) | 0.1 mm |
| \(dt_{\text{min}}\) | 0.017 s |
| \(n_z\) | 2 |
| \(t_{dz}\) | 2 s |
| **Calculated** | **Calculated** |
| \(v_{\text{max}}\) | 5.88 mm/s (Eq. 3.3) |
| \(t_d\) | 15.6 s (Eq. 3.4) |

In order to determine the measurement time for a meander scan, it is necessary to consider scanning distance along each linear scan. For circular apertures, each line scan has different travel distances. Therefore, time required for each line scan has to be calculated and summed up to determine the total measurement time. The total measurement time for a meander scan can be calculated with

\[
t_{\text{total}} = \sum_{i=1}^{N} \frac{s_{\text{dist}}(i)}{v_{\text{max}}} \cdot n_z + t_{dz}(N - 1) + t_{dz}(n_z - 1)
\]

where \(N\) is the number of linear scans over the circular aperture and \(t_{dz}\) is the time required between two linear scans in \(x\) axis. It should be noted that linear scans are performed along \(y\) axis. The scanning distance along one linear scan is given as

\[
s_{\text{dist}} = 2 \cdot \sqrt{r^2 - (r - i \cdot dy)^2}
\]
where \( r \) is the radius of the measurement field. An example of the calculation of the measurement time for a meander scan is given in Table 3.2.

**Table 3.2.** Measurement time required for an example meander scan.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D )</td>
<td>40 mm</td>
</tr>
<tr>
<td>( dy )</td>
<td>0.25 mm</td>
</tr>
<tr>
<td>( dt_{\text{min}} )</td>
<td>0.017 s</td>
</tr>
<tr>
<td>( n_z )</td>
<td>2</td>
</tr>
<tr>
<td>( t_{dx} )</td>
<td>0.3 s</td>
</tr>
<tr>
<td>( t_{dz} )</td>
<td>2 s</td>
</tr>
<tr>
<td>Calculated</td>
<td></td>
</tr>
<tr>
<td>( v_{\text{max}} )</td>
<td>14.7 mm/s (Eq. 3.3)</td>
</tr>
<tr>
<td>( t_d )</td>
<td>733 s (Eq. 3.5)</td>
</tr>
</tbody>
</table>

It can be seen from Eq. 3.5 that the number of \( z \) positions \( n_z \) used in the measurement has a major effect on the measurement time. This is basically because at each \( z \) position a complete meander scan should be performed. If three \( z \) positions \( (n_z) \) were used in the example shown in Table 3.2, then the total measurement time would have been increased from 733 seconds to 1076 seconds. Therefore, it is more practical to use two \( z \) positions for meander scan measurements in order to keep the measurement time as short as possible. It should be mentioned that these calculations consider only the scanning process of the measurement. The preparation time for the measurements, which includes the calibration and the alignment of the test lens, is not considered. The duration of the preparation time depends on the experience and ability of the operator of the measurement setup.

### 3.3. Structure of the analysis software

After a measurement is performed, the measurement data is saved to a file and it is ready to be analyzed. For this purpose, analysis software has been written in Matlab. Simplified structure of the analysis software is illustrated in Fig. 3.8. At first, the measurement data file is read and the data is processed using the input parameters already given into the controlling unit software. As can be seen from Fig. 3.8, the main structure of the software is divided into two paths. One path leads to the determination of the optical properties of the lens and the other path leads to the retrieval of the aspherical surface profile of the lens. Calculated optical properties are the focal length, the ray aberrations and the wave aberrations. For the latter one, both modal (Zernike polynomials) and zonal methods are used. Moreover, statistical data of the wavefront aberrations are determined (See detailed flowchart in Appendix B). In the ASR routine, aspherical surface profile of the lens is reconstructed. Lens design data is used in both
ASR routine and in the calculation of the statistical surface data. This is basically required for the comparison of design aspherical surface and the best-fit radius aspherical surface profiles.

![Diagram of measurement data processing](image)

**Figure 3.8.** Simplified structure of the analysis software. Measurement data are used in two sub-routines; Optical properties and ASR. Both sub-routines deliver statistical data of the results. Lens design data is used in ASR routine as well as in the calculation of the statistical surface data.

### 3.4. Calibration of the measurement setup

Accurate determination of the misalignments and the angular errors between the reference coordinate system and the observation (camera) planes is necessary for compensating the systematic errors in the ERT setup. For this purpose, a calibration routine has been developed. Note that the calibration has to be performed without the test lens in the measurement plane, because the aim is to eliminate the systematic errors in the measurement system independent from the misalignment errors of the lens.

The schematic layout of the reference and the camera coordinate systems are shown in Fig. 3.9. For simplicity, this figure is illustrated in two axis; y and z. Initially, the origin of the reference coordinate system \( O \) has to be identified. In the ERT setup, the center position feedback values \((0,0)\) from the controller of the XY translation stages are considered to be origin of the reference coordinate system \((x_0, y_0, 0)\). For the \( z \) component of the origin \( O \), the nearest position of the Z translation stage to the lens holder is considered to be at \((z_0 = 0)\). Furthermore, the position feedback values of the XY and Z translation stages controllers are assumed to describe the reference coordinate system \((x_0, y_0, z_0)\). The coordinates of the test ray at the camera planes corresponds to the measured spot centroid.
position at each \( z \) plane and they are denoted by \((S_x, S_y, 0)\). Here, the \( z \) term of the measured spot centroid positions is taken zero since they describe the local ray coordinates at the camera planes.

\[
(x_0, y_0, z_0 = 0)
\]

\[
(x_0, y_0, z_0 = \Delta z)
\]

\[
(S_x^z, S_y^z, 0)
\]

\[
(S_x^z, S_y^z, 0)
\]

**Figure 3.9.** Schematic layout of the orientations of the reference coordinate system and to the camera planes.

The Euler angles \((\phi, \theta, \psi)\) specify the misalignment of the camera planes with respect to the reference coordinate system independent from the \( z \) position of the camera. The angles \((\xi_x, \xi_y)\) describe the angular error along the \( z \) direction in \( x \) and \( y \) axis.

In general, five different misalignment parameters are identified to specify the orientation of the reference coordinate system with respect to the local coordinates of the camera planes. The Euler angles \((\phi, \theta, \psi)\) are used for describing the misalignment of the camera planes with the reference coordinate system (only \( \phi \) is shown in Fig. 3.9). These angles are assumed to be independent from the \( z \) location of the camera in the coordinate system. In addition, the angular misalignments of the camera plane along the \( z \) direction are represented by \((\xi_x, \xi_y)\).

Here, \( \xi_x \) and \( \xi_y \) describe the angular misalignments in \( x \) and \( y \) axis respectively.

First, the relationship between the reference coordinate system and the local coordinates of the test ray at the camera position \( z_1 \) can be written by rearranging the Eq. 2.19 as

\[
\begin{bmatrix}
  x_0 \\
  y_0 \\
  0
\end{bmatrix}
= \begin{bmatrix}
  S_x^z \\
  S_y^z \\
  0
\end{bmatrix}
\]

(3.7)

where \( R \) is the rotation matrix specified by the Euler angles \((\phi, \theta, \psi)\). In Eq. 3.7, no offset values are used since the center position of the camera is located at the origin \( O \) of the
reference coordinate system. In the same manner, the reference coordinates with respect to the
test ray coordinates at the camera position \( z_2 \) is given by

\[
\begin{bmatrix}
x_0 \\
y_0 \\
\Delta z
\end{bmatrix} = R^T \begin{bmatrix}
S_x^z \\
S_y^z \\
0
\end{bmatrix} \begin{bmatrix}
\rho_x \\
\rho_y \\
\rho_z
\end{bmatrix}
\]

(3.8)

where \((\rho_x, \rho_y, \rho_z)\) define the offset coordinates between the center coordinate of the camera
at position \( z_2 \) and the origin \( O \) of the reference coordinate system. These offset coordinates
can be written from the geometry as

\[
\begin{bmatrix}
\rho_x \\
\rho_y \\
\rho_z
\end{bmatrix} = \begin{bmatrix}
tan \xi_x \cdot \Delta z \\
tan \xi_y \cdot \Delta z \\
\Delta z
\end{bmatrix}
\]

(3.9)

By solving the Eq.s 3.7 and 3.8 using a least squares fitting method, the angles \((\phi, \theta, \psi, \xi_x, \xi_y)\)
can be determined. Then using these angles, one can specify the coordinates of the test ray at
any \( z_k \) position of camera in terms of the reference coordinate system by

\[
\begin{bmatrix}
x_0 \\
y_0 \\
\Delta z_k
\end{bmatrix} = R^T \begin{bmatrix}
S_x^z \\
S_y^z \\
0
\end{bmatrix} \begin{bmatrix}
tan \xi_x \cdot \Delta z_k \\
tan \xi_y \cdot \Delta z_k \\
\Delta z_k
\end{bmatrix}
\]

(3.10)

Calibration procedure can be described by the following steps:

- Definition of the camera sensitive area (square aperture)
- Definition of the scan positions inside the camera sensitive area
- Definition of the \( z \) positions of the camera (\( z_i \) position is always taken as the \( z_0 = 0 \) of
  reference coordinate system)
- Setting the coordinate of the test ray to 0 at the origin of the reference coordinate
  system
- Scanning the camera aperture at given \( x, y \) and \( z \) positions
- Calculation of the angles \((\phi, \theta, \psi, \xi_x, \xi_y)\) using least squares fitting
3.5. Error Analysis

Evaluating the measurement uncertainty plays a critical role of understanding the capabilities and limitations of the measurement systems. In this section, error sources in the ERT setup will be in detail investigated and discussed. As theoretical investigation, the Law of Propagation of Uncertainty (LPU) will be used and the effect of the uncertainty sources of different parameters to the overall uncertainty will be predicted. As an alternative, Monte Carlo analysis will be presented and the limitations of the measurements will be discussed.

3.5.1. Propagation of uncertainty – slope measurements

Consider an output quantity $T[p_1, \ldots, p_N]$ which is defined in terms of set of input quantities $p_i$. In LPU method, variance $V$ of the output quantity $T$ is defined as [77]

$$V_T = u_T^2 = \sum_{i=1}^{N} c_i^2 \cdot u_{p_i}^2 + 2 \cdot \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} c_i c_j u_{p_i} u_{p_j}$$

(3.11)

where $u_{p_i}$ denotes the uncertainty of the input quantity $p_i$. The coefficients $c_i$ are the partial derivative of the model function $M$ with respect to the measured quantity $p_i$, which are given by

$$c_i = \left| \frac{\partial T}{\partial p_i} \right|$$

(3.12)

The first term of Eq. 3.11 describes the contribution of individual quantities $p_i$ to the overall uncertainty of the output quantity $T$. In the second term, uncertainty sources of the correlated variables are indicated.

In order to apply the LPU method to ERT setup, output quantity $T$ (slope of the ray) should be described in terms of the individual measured quantities (See Fig. 3.10). One dimensional simplified description of $T$ for two $z$ planes can be written as

$$T = \frac{S^1_y - S^2_y}{z_1 - z_2}$$

(3.13)

Here, $T$ is the calculated slope of the ray. $S^1_y$ and $S^2_y$ are the obtained centroid positions at two different $z$ positions ($z_1$ and $z_2$) of the camera along propagation direction of the ray. Note that, this theoretical investigation is limited to evaluation of the uncertainty in the slope measurements. Therefore, errors introduced by the translation stages will be neglected at this point.
Applying the method described by Eq. 3.11 to the model function in Eq. 3.13, one leads to

\[
ur^2 = c_1^2 \cdot u_{S_1}^2 + c_2^2 \cdot u_{S_2}^2 + c_3^2 \cdot u_{S_z}^2 + c_4^2 \cdot u_{z_1}^2 + \ldots
\]

\[
2 \left[ c_1 c_2 u_{S_1 z_1} + c_1 c_3 u_{S_1 z_2} + c_1 c_4 u_{S_1 z_3} + c_2 c_3 u_{S_2 z_1} + c_2 c_4 u_{S_2 z_2} + \ldots \right]
\]

(3.14)

Then, using Eq. 3.12, coefficients are found as

\[
c_1 = \frac{1}{z_1 - z_2}
\]

\[
c_2 = \frac{1}{z_1 - z_2}
\]

\[
c_3 = \frac{S_1^2 - S_2^2}{(z_1 - z_2)^2} \Rightarrow \frac{T}{(z_1 - z_2)}
\]

\[
c_4 = \frac{S_1^2 - S_2^2}{(z_1 - z_2)^2} \Rightarrow \frac{T}{(z_1 - z_2)}
\]

The correlated quantities in the model function of ERT setup are \((S_y', z_1')\) and \((S_y'^2, z_2')\). The input quantities \((S_y^1, S_y^2), (S_y', z_1), (S_y'^2, z_2)\) and \((z_1, z_2)\) are statistically independent, therefore, their covariances are taken as zero. It can be assumed that the uncertainties of the centroid positions \(u_{S_y}\) and \(u_{S_z}\) are identical since they basically result from the camera noise, not from the camera location. A similar assumption holds as well for the uncertainties in z positioning \(u_{z_1}\) and \(u_{z_2}\), because expected uncertainty of the z positioning system would be approximately identical along the complete travel path of the z positioning system. Then, using the notations \(u_{S_y}, u_z\) and \(\Delta z\) for difference between two camera positions \((z_1 - z_2)\), Eq. 3.14 reduces to
\[
\begin{align*}
    u_T^2 &= 2 \cdot \left( \frac{1}{\Delta z} \right)^2 \cdot u_{S_y}^2 + 2 \cdot \left( \frac{T}{\Delta z} \right)^2 \cdot u_z^2 + 2 \cdot \left( \frac{T}{(\Delta z)^2} \right) \cdot u_{S_y,z}^2 \\
    \text{(3.15)}
\end{align*}
\]

Here, it can be seen that the uncertainty of the slope measurements \( u_T \) is dependent on \( \Delta z, T \) (the actual absolute value of the slope) and uncertainties of each input quantity. It is obvious that \( u_T \) is decreased when \( \Delta z \) is increased. Furthermore, when the absolute value of the slope \( T \) is increased, the uncertainty of slope measurements \( u_T \) increases.

Uncertainty of slope measurements described by Eq. 3.15 is simulated for different slope values of \( T \) and different \( \Delta z \) values. Fig. 3.11 illustrates the calculated corresponding uncertainty of slope estimations. For this simulation standard deviation of the centroid calculation \( u_{S_y} \) is taken as 80 nm. Uncertainty of motor positioning for \( z \) axis \( u_z \) is taken as 100 nm. Covariance \( u_{S_y,z} \) is calculated by

\[
    u_{S_y,z} = \text{cov}(S_y, z) = corr(S_y, z) \cdot u_{S_y} \cdot u_z \\
    \text{(3.16)}
\]

Here, the correlation coefficient is assumed to be +1, since the simulations are performed for positive slopes. This means that positive deviations in the \( z \) position would lead to positive deviations for \( S_y \) position. For negative slopes, the correlation coefficient should be taken -1.

As can be seen from Fig. 3.11, the uncertainty of slope measurements \( u_T \) is strongly dependent on the \( \Delta z \). \( u_T \) dramatically decreases when \( \Delta z \) is increased. In order to achieve 5 \( \mu \)rad uncertainty, approximately \( \Delta z \) of 25 mm is required. On the other side, the effect of the absolute value of the slope \( T \) is less compared to that of \( \Delta z \). For the given parameters, when the slope \( T \) is increased from 0 to 1 (0 to 0.7853 rad or 0° to 45°), \( u_T \) increases only by 0.168 \( \mu \)rad.
In order to investigate the effects of the uncertainties of the input parameters $u_z$ and $u_{s_z}$ on the uncertainty in the slope measurements $u_T$, two series of simulations are performed. In both simulations, actual slope $T$ is taken as $0.4536$ rad ($22.5\degree$). In the first set of simulations, $u_z$ is kept at 80 nm and five different $u_{s_z}$ values between 25 nm and 300 nm are used. The simulations are also performed for different $\Delta z$ values. For the second set, same methodology is applied but keeping $u_{s_z}$ at 80 nm and varying the $u_z$ values. As can be seen from the results of the simulations shown in Fig. 3.12, the effect of the uncertainty in centroid positioning $u_{s_z}$.
is more dominant compared to that in \( z \) positioning. When \( u_{S_y} \) is increased from 25 nm to 300 nm, \( u_T \) increases approximately from 12 \( \mu \)rad to 85 \( \mu \)rad. On the other side when \( u_z \) is varied in the same interval, \( u_T \) increases from 23 \( \mu \)rad to 45 \( \mu \)rad. This indicates that high values of \( u_{S_y} \) can significantly degrade the performance of slope measurements. At this point, one important property of the ERT method plays a critical role. It is the flexibility of using different \( \Delta z \) values in the measurement system. Even for high uncertainties of \( u_z \) and \( u_{S_y} \), one can simply use large \( \Delta z \) values in the measurement system and decrease \( u_T \) well below 10 \( \mu \)rad.

![Figure 3.12](image)

**Figure 3.12.** Effect of the uncertainty of centroid measurement \( u_{S_y} \) and uncertainty in \( z \) positioning \( u_z \) on the uncertainty in calculated slope \( u_T \) for \( T = 0.4636 \) rad. a) Uncertainty \( u_T \) with respect to \( \Delta z \) for different \( u_{S_y} \) values \((u_z = 80 \text{ nm})\) b) Uncertainty \( u_T \) with respect to \( \Delta z \) for different \( u_z \) values \((u_{S_y} = 80 \text{ nm})\).

### 3.5.2. Monte Carlo simulations of the experimental ray tracing

In order to evaluate the uncertainty of the complete ERT setup, Monte Carlo simulations are performed. A computer algorithm is used to generate sequence of values for each input parameter \((y, S^1_y, S^2_y, z_1, z_2)\) with a predefined point distribution function. This random process is performed with an estimated standard uncertainty for each input parameter. In these simulations uncertainty in the scan positions \( u_z \), which are introduced by the XY translation stages, are also included. A synthetic test aspherical lens with a defined geometry is generated. Parameters used for the simulation of the lens are shown in Table 3.3.
Table 3.3. Aspherical surface parameters of the lens used in the Monte Carlo simulation.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$R$</th>
<th>$k$</th>
<th>$n$</th>
<th>$t$</th>
<th>$A_2$</th>
<th>$A_4$</th>
<th>$A_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>32.77</td>
<td>-1</td>
<td>1.655743</td>
<td>9.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The complete process of the Monte-Carlo simulation can be simplified by the following steps;

1 – Tracing rays at given scan positions $y$ through the defined lens geometry (Simulation is based on ray tracing method explained in Sec. 2.2)

2 – Calculation of the intercepts of the simulated transmitted ray with the different $z$ positions of the detector and determination of slopes $T_y$ of the transmitted rays. (This step includes no error)

3 – Generation of error at each scanning position by $y + \delta y$

4 – Introducing a randomly generated error to each $z$ position $z + \delta z$ and tracing the rays through the lens again

5 – Adding a randomly generated error for centroid estimation at the camera $S_y + \delta S_y$

6 – Calculation of the slopes of the transmitted rays using the values introduced at step 4 and 5.

7 – Repetition of the simulation $N$ times

8 – Comparison of the slope values calculated at step 2 and the mean slope errors over the complete aperture of the lens are calculated at step 7.

Uncertainty of the slope measurements in the Monte-Carlo simulation are calculated by

$$u_T = \sqrt{\frac{1}{N} \sum_{i=1}^{N} T[y + \delta y_i, S_{y_i} + \delta S_{y_i}^1, S_{y_i}^2 + \delta S_{y_i}^2, z_i + \delta z_i, z, z + \delta z_z]}$$

(3.17)

The simulations are performed for different $\Delta z$ values as in the LPU method. Here, the difference from LPU method is that the effect of the uncertainty in scanning positions $u_y$ on the uncertainty of slope measurements $u_T$ can also be investigated. For this purpose, simulations are performed for fixed $u_{S_y}$ and $u_z$ values and for various $u_y$ values. Results of the simulations are shown in Fig. 3.13. At the first glance, the agreement between the results of LPU and Monte-Carlo methods can be seen. However, uncertainty in scan positioning $u_y$ has a different impact on $u_T$ compared to $u_{S_y}$ and $u_z$. The effect of an increase in $u_y$ values is more dominant for greater $\Delta z$ values. For $\Delta z = 50$ mm, the change in $u_y$ from 25 nm to 300 nm
increases the $u_T$ approximately by factor of two. On the other side for $\Delta z = 5\, \text{mm}$, the same change is almost negligible.

![Graph showing the effect of uncertainty in scan positioning system on ray slope measurements.](image)

**Figure 3.13.** Effect of the uncertainty of the scan positioning system $u_y$ on the uncertainty of ray slope measurements $u_T$ for $u_y = 80\, \text{nm}$ and $u_z = 80\, \text{nm}$.

### 3.5.3. Uncertainty analysis of the experimental ray tracing setup

Centroid error of an image spot fluctuates from air turbulence, optic vibration, instability in laser intensity, and noise in such detector units as the CCD camera and the image processor [78]. Error in centroid estimation can be determined by capturing a large amount of images and calculating the centroid position of each image. This procedure must be done in identical condition for each measurement. This results to the standard deviation of centroid estimation by

$$u_S = \sqrt{\frac{1}{N} \sum_{n=1}^{N} (S_{y_{n}} - \bar{S}_y)}$$  \hspace{1cm} (3.18)

where $N$ indicates the number of measurements. The camera exposure time can be programmed to obtain different intensity levels. Table 3.4 presents the uncertainty of centroid estimation with respect to different exposure time, hence different intensity levels. 1000 measurements were performed after a warm up time of two hours.

**Table 3.4.** Uncertainty of centroid estimation. Repeatability measurements performed in identical conditions for various exposure times of the camera.

<table>
<thead>
<tr>
<th>Normalized Intensity ($I/I_0$)</th>
<th>Exposure Time (µs)</th>
<th>$u_{Sy}$ (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>0.2</td>
<td>110</td>
</tr>
<tr>
<td>0.5</td>
<td>0.3</td>
<td>80</td>
</tr>
<tr>
<td>0.7</td>
<td>0.4</td>
<td>62</td>
</tr>
<tr>
<td>0.94</td>
<td>0.5</td>
<td>47.6</td>
</tr>
</tbody>
</table>
Correlation between the normalized intensity with the uncertainty of the centroid estimation can clearly be seen in Table 3.4. Normalized intensity value \( I/I_0 \) 0.4, delivers an uncertainty of 110 nm, while uncertainty can be improved approximately by a factor of 2 when exposure time is increased to 0.5 \( \mu \)s. One should be careful about setting the exposure time of the camera. Exposure time must also set to a minimum value as much as possible because of the fact that the spot on the camera shifts in the direction of the movement of the translation stages. This would result into elliptical shaped intensity distributions which should be as well calculated in the error budget. However, this is a systematic error, not a random error; therefore it will not be considered in the simulations presented here. Furthermore, this error can be compensated by estimating the shift of the spot for a given speed of the translation stages and the exposure time of the camera.

The positioning uncertainty of the XY translation stages in the measurement setup is determined by collecting large amount of position data in identical conditions. Sensitive area of the camera is scanned by the light source mounted on the XY translation stages. The measurements are performed with a test lens in the measurement system. Linear scans are done along \( y \) axis and in an interval of -3.5 mm to +3.5 mm. The distance between each scan point is taken as 100 \( \mu \)m and at each point the location of the centroid \( S_y \) is calculated. 100 consecutive scans are performed under same conditions. At the end of the measurements, standard deviation of the measured centroids \( u_{S_y} \) for each \( y \) position of the XY positioning system is calculated. The results are shown in Fig. 3.14. It can be seen that \( u_{S_y} \) fluctuates around its mean value 129 nm.

![Figure 3.14](image-url)  
Figure 3.14. Uncertainty of the measured centroid positions with respect to the scan positions \( y \).
One important point in these measurements is that the uncertainty of the measured centroid positions consists of the uncertainty of XY positioning system $u_y$ and the uncertainty of centroid estimation. In other words, the measured data set carries the information of both of the uncertainty sources. Here, the objective is to extract the uncertainty information of XY translation stages. Assuming that both uncertainties are statistically independent from each other, the uncertainty of the measured centroid positions can be written as

$$u_{S_y}^2 = u_{S_y}^2 + u_{y}^2$$  \hspace{1cm} (3.19)

Here, in order to avoid confusion, uncertainty of measured centroid positions and uncertainty of centroid estimation are denoted by $u_{S_y}$ and $u_{y}$ respectively. $u_{y}$ can be used as a moderate value of 60 nm from Table 3.4. Then, uncertainty of XY translation stages is calculated from Eq. 3.19 as approximately 114 nm. These values can be used for the same simulations presented in the previous section which indicates the uncertainty of the ERT setup realized in this work (See Fig. 3.15). Minimum uncertainty of slope measurements $u_T$ is calculated as 3.248 µrad for $\Delta z = 50$ mm.

**Figure 3.15.** Uncertainty of slope measurements $u_T$ calculated for the realized ERT setup using the measured individual uncertainties.

### 3.5.4. Wavefront sensitivity

The wavefront sensitivity of the ERT setup is defined by

$$W_{\text{min}} = u_T \cdot dy$$  \hspace{1cm} (3.20)

where $dy$ is the spatial distance between two sampling points. This property defines the minimum detectable wavefront change in a given spatial region. As $u_T$ depends on the $\Delta z$ used in the measurements and $dy$ is scalable by the XY translation stages, wavefront sensitivity of
ERT setup is an adjustable property. For example, if \( dy \) is used as 100 µm and \( u_r \) is 3.248 µrad (as given in the previous section), the wavefront sensitivity of the ERT setup for \( \Delta z = 50 \text{ mm} \) is found as 3.2 nm.

The comparison of the wavefront sensitivities of the ERT and SHSs is straightforward. As defined already in Eq. 1.14, wavefront sensitivity of SHSs depends strongly on the focal length of the microlenses. Employing longer focal lengths would increase the dynamic range of the SHSs; however, they are generally designed in the range of 5 to 10 mm. The reason is that long focal lengths significantly limit the dynamic range. However, ERT is much more flexible compared to SHSs. The main advantage is the possibility of changing the wavefront sensitivity by simply setting different camera positions by the Z translation stage. Furthermore, a large travel range of Z translation stage allows up to 120 mm \( \Delta z \) values, which is in between 10 to 20 times longer than typical focal lengths used in the SHSs.

### 3.5.5. Dynamic range

In order to define the dynamic range of ERT, the maximum measurable slopes should be determined. This is also called the angular dynamic range. Rocktäschel et. al [27] had investigated in detail the dynamic range of the SHS in comparison with interferometric measurements for aspherical lenses. The same methodology is applied in this work. Fig. 3.16 depicts the geometry of the measurement setup for the calculation of the maximum measurable wavefront slope.

![Figure 3.16. Maximum measurable slope in the ERT setup.](image)

Maximum measurable slope in the ERT setup is given as
where \( m \) is the total length of the active area of the camera and \( d_{\text{spot}} \) is the diameter of the spot on the sensor. Since \( d_{\text{spot}} < m \), the relationship can be simplified to \( T_{y_{\text{max}}} = m / \Delta z \).

Theoretically, \( \Delta z \) can be set to the minimum incremental motion value of the Z translation stage. This corresponds to 200 nm in the demonstrated measurement setup in this work. As can be seen from Eq. 3.21, for very small \( \Delta z \) values, extremely steep rays can be measured. The dynamic range of the ERT setup with respect to \( \Delta z \) is shown in Fig. 3.17. Here, active area of the camera \( m \) is assumed to be 10 mm. The problem is, as discussed in Sec. 3.5.1, for very short \( \Delta z \) values the uncertainty of the slope measurements \( u_T \) increases rapidly. This is the trade-off between dynamic range and the uncertainty of slope measurements. But, it should be noted that such steep slopes (more than 1 rad) are not realistic in lens design. For example, a lens with a \( f \)-number of 1, would generate a maximum ray slope of 0.4636. Therefore, it is generally not necessary to use extremely short \( \Delta z \) values. On the other side, by utilizing a camera with a larger active area \( m \), one can increase the dynamic range of the ERT setup.

\[
T_{y_{\text{max}}} = \tan \alpha_{\text{max}} = \frac{m - d_{\text{spot}}}{\Delta z} \quad (3.21)
\]

---

**Figure 3.17.** Dynamic range of ERT setup is variable. It depends on the \( \Delta z \) used in the measurements. Active area of the camera \( m \) is 10 mm.

As mentioned in Chap. 1, The ERT setup measures the slopes of the rays, thereby the first derivative of the wavefront. By taking into account only the rotationally symmetric components of the wavefront described by up to 13\(^{\text{th}}\) term Zernike polynomials, the first derivative of the wavefront can be written as

\[
\frac{\partial W(y)}{\partial y} = c_5 \cdot \left[ 4 \frac{y}{r_0^2} \right] + c_{13} \cdot \left[ 12 \left( 2 \frac{y^3}{r_0^4} - \frac{y}{r_0^2} \right) \right] \quad (3.22)
\]
where \( r_0 \) is the radius of the aperture of the lens. Inserting the equation for the maximum measurable slopes of rays by the ERT setup into Eq. 3.22, the maximum measurable 5th and 13th terms Zernike coefficients can be calculated by

\[
\begin{align*}
  c_5^{\text{max}} &= \frac{m r_0^2}{\Delta z \cdot 4 y} \\
  c_{13}^{\text{max}} &= \frac{m r_0^4}{\Delta z \cdot 12 \left( 2 y^3 - 3 y r_0^2 \right)}
\end{align*}
\]  

(3.23)  

(3.24)

where \( c_5 \) (Focus shift) represents the main curvature of the aspheric wavefront and \( c_{13} \) (3rd order spherical aberration) is the aspheric component of the wavefront. Consider now a lens with radius \( r_0 \) of 25 mm. For a measurement with \( \Delta z = 20 \) mm, using the Eq.s 3.23 and 3.24, the maximum measurable Zernike coefficients \( c_5^{\text{max}} \) and \( c_{13}^{\text{max}} \) are calculated as 3.125 mm and 1.041 mm respectively. The wavefronts constructed from these Zernike coefficients are illustrated in Fig. 3.18. Under these conditions, the ERT setup, without the use of a null lens, can measure spherical wavefronts with P-V of 6.25 mm. On the other side, P-V of the aspheric component is limited to 1.561 mm. Rocktäschel calculated the maximum measurable P-V spherical wavefront with SHSs as 188 \( \mu \)m. This value is decreased to 13.5 \( \mu \)m for P-V aspherical wavefront. For interferometers, same parameters are 22 \( \mu \)m and 5.55 \( \mu \)m respectively. This example validates the potential of the ERT method in measurement of aspherical lenses. ERT method offers a high dynamic range compared SHSs and interferometers.

**Figure 3.18.** Maximum measurable wavefront by the ERT setup a lens with \( r_0 = 25 \) mm. \( \Delta z \) is taken 20 mm and active area of the camera \( m \) is 10 mm.
Overlapping of spot positions on the camera is the limiting factor for SHSs. This is not a problem in ERT, since the rays are traced sequentially. Therefore, there is only a single spot on the camera at each scan position. However, sequential ray tracing increases the time duration significantly for a single measurement in the ERT method compared to SHSs.

### 3.5.6. Analysis of the Zernike polynomials for focal length calculations

In this section, an error analysis of the determination of the focal length using the Zernike polynomials will be given. The main idea of the error analysis is to simulate the transmitted wavefronts with different parameters and to analyze the calculated focal length by the procedure given in Sec. 2.7. Initially, it is assumed that the transmitted wavefronts $W_a$ do not have aberrations. Thereby, Gaussian reference sphere $S$ is simply equal to the $W_a$ (See Fig. 2.6). Furthermore, as mentioned in Sec. 2.4, in ERT radius $R$ of the Gaussian reference sphere equals to the effective focal length of the lens. Hence, Gaussian reference spheres are taken into consideration for the simulations. Different Gaussian reference spheres have been synthetically generated by a computer algorithm. The parameters that describe the Gaussian reference sphere $S$ are the $f$-number of the lens (diameter and focal length) and the number of sampling points $N$. Simulated derivatives of the Gaussian reference sphere are then fitted to the derivatives of the Zernike polynomials and the corresponding Zernike coefficients are determined. Finally using the Eq. 2.56, the focal length of the lens is calculated.

As an initial investigation, a Gaussian reference sphere $S$ is generated with a $f$-number of 1 using 5025 sampling points $N$. This surface is fitted to the Zernike polynomials from 2nd order up to 10th order. Apparently, resulting non-zero Zernike coefficients are simply the Zernike terms which contain paraxial terms. These are the 5th, 13th, 25th, 41st and 61st Zernike terms. For each case, error $\varepsilon_f$ between the simulated and the calculated focal length are determined. The results are presented in Table 3.5. It can be seen that further increasing the number of Zernike orders used in the fitting process decreases the error $\varepsilon_f$. This result is quite reasonable because using more terms increases the accuracy of representation of the surface.

<table>
<thead>
<tr>
<th>Fit order up to</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_f$ [%]</td>
<td>8.87684</td>
<td>0.96627</td>
<td>0.0878</td>
<td>0.007717</td>
<td>0.0006553</td>
</tr>
</tbody>
</table>

Calculated individual Zernike coefficients for each case are shown in Table 3.6. Here, it can be seen that when the number of fit orders are increased, coefficients of the lower terms are...
affected. This proves that the Zernike polynomials are not orthogonal for a discrete data set. However, the change in the lower term coefficients, when higher order terms are included, is negligible.

<table>
<thead>
<tr>
<th>Zernike Coefficient No.</th>
<th>Fit up to 2\textsuperscript{nd} order</th>
<th>Fit up to 4\textsuperscript{th} order</th>
<th>Fit up to 6\textsuperscript{th} order</th>
<th>Fit up to 8\textsuperscript{th} order</th>
<th>Fit up to 10\textsuperscript{th} order</th>
</tr>
</thead>
<tbody>
<tr>
<td>c\textsubscript{5}</td>
<td>0.137176992</td>
<td>0.13383645</td>
<td>0.133836473</td>
<td>0.133836474</td>
<td>0.133836474</td>
</tr>
<tr>
<td>c\textsubscript{13}</td>
<td>-</td>
<td>0.003344244</td>
<td>0.003199228</td>
<td>0.003199231</td>
<td>0.003199231</td>
</tr>
<tr>
<td>c\textsubscript{25}</td>
<td>-</td>
<td>-</td>
<td>0.000145196</td>
<td>0.000137726</td>
<td>0.000137726</td>
</tr>
<tr>
<td>c\textsubscript{41}</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>7.47831E-06</td>
<td>7.06008E-06</td>
</tr>
<tr>
<td>c\textsubscript{61}</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>4.18753E-07</td>
</tr>
</tbody>
</table>

Another important parameter that effects the focal length determination is the $f$-number of the lens. This parameter indicates the curvature of the Gaussian reference sphere. Gaussian reference spheres with different $f$-numbers are simulated and the corresponding focal lengths are calculated. The results shown in Table 3.7 indicates that the higher the $f$-number of the lens, the higher the error $\varepsilon_f$. It should be noted that the absolute value of the focal length is not decisive parameter for the $\varepsilon_f$. It is rather the curvature of the Gaussian reference sphere.

<table>
<thead>
<tr>
<th>$f$-number</th>
<th>$f$/1</th>
<th>$f$/2</th>
<th>$f$/4</th>
<th>$f$/8</th>
<th>$f$/16</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_f$ [%]</td>
<td>0.0878</td>
<td>9.687E-04</td>
<td>1.397E-05</td>
<td>2.1416E-07</td>
<td>3.3E-09</td>
</tr>
</tbody>
</table>

As in all fitting processes, number of sampling points plays a critical role concerning the accuracy of the process. Here, simulations are performed for 81, 317, 1257 and 5025 sampling points $N$. These values are obtained from 11x11, 21x21, 41x41 and 81x81 square grid scanning points. Simulated lens has a $f$-number of 1 and up to 6\textsuperscript{th} order Zernike polynomials are used in fitting process. Fig. 3.19 illustrates the deviations from the simulated $S$ and the reconstructed surface $S_f$ for each case. Deviations are plotted over the normalized $y$ coordinate. It can easily be seen that with low sampling points, the resolution gets poorer and the deviations are increased. For a good representation of the $S$, at least 41x41 grid scanning points should be used.
3.5.7. Monte Carlo simulations of the aspherical surface retrieval

Simulated slopes of the rays that generated by the lens parameters given in Table 3.3 are used to analyze the effect of the different uncertainty sources in the ERT setup on the calculated best-fit radius values from the ASR routine. For this purpose, simulations are performed by the procedure described in Sec. 3.5.2 and calculated slopes of the transmitted rays for various situations are delivered to the ASR routine and the best-fit radius of the aspherical surface of the lens is determined. Each simulation is repeated 500 times and uncertainty in the best-fit radius \( u_R \) is calculated. Table 3.8 simplifies the parameters used in the simulations and the calculated best-fit radius values are presented in Fig. 3.20. The analysis is handled in four different cases. In case 1, all individual uncertainties \( u_z, u_S, u_y \) are used as 100 nm. In other three cases, individual uncertainties systematically increased to 300 nm one by one. The main aim is to compare the influences of the uncertainties of each component in the ERT setup on the calculated best-fit radius values.
Table 3.8. Uncertainty values that are used in Monte Carlo simulations in combination with the ASR routine. Calculated best-fit radius value for each case are listed at the right column.

<table>
<thead>
<tr>
<th></th>
<th>$u_z$</th>
<th>$u_y$</th>
<th>$u_z$</th>
<th>$u_R$(best-fit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>100 nm</td>
<td>100 nm</td>
<td>100 nm</td>
<td>187 nm</td>
</tr>
<tr>
<td>Case 2</td>
<td>300 nm</td>
<td>100 nm</td>
<td>100 nm</td>
<td>435 nm</td>
</tr>
<tr>
<td>Case 3</td>
<td>100 nm</td>
<td>300 nm</td>
<td>100 nm</td>
<td>346 nm</td>
</tr>
<tr>
<td>Case 4</td>
<td>100 nm</td>
<td>100 nm</td>
<td>300 nm</td>
<td>223 nm</td>
</tr>
</tbody>
</table>

As the results in Fig. 3.20 indicate, best-fit radius values fluctuate around 32.77 mm. This agrees with the radius of curvature value of the lens used in the simulation. It is possible to say that the ASR routine works properly; it calculates the intended radius of curvature value. On the other side, influence of the XY Z translation stages and the centroid calculation uncertainties show differences. In case 2, $u_R$ is increased by more than a factor of 2 to 435 nm in comparison to case 1. However, in case 3 and in case 4 $u_R$ is only increased by a factor of 1.8 and 1.2 respectively. These results show that the selection of Z translation stage is extremely important in the ERT setup. It has the major contribution on the calculated best-fit radius error by the ASR routine.

![Figure 3.20](image-url)

**Figure 3.20.** Comparison of four different cases for certain uncertainty sources for ERT setup. At each simulation best-fit radius value is calculated by the ASR routine.
3.5.8. Systematic error sources of the aspherical surface retrieval

The ASR approach is based on the assumption that the second surface, the refractive index and the thickness of the aspherical lens are known. These parameters are used in the surface reconstruction engine as input parameters for numerical ray tracing. However, obviously there would be deviations from the actual values of these parameters from the measured ones. The aim in this section is to determine the individual effects of these errors on the reconstructed aspherical surface. Note that the errors will be handled as systematic errors, not statistical errors.

First, the effect of a systematic error on the input parameter radius of curvature of the second surface \( R_2 \) will be discussed using the Lensmaker equation. The Lensmaker equation is the mathematical description of the focal length of the lens in paraxial approximation with respect to both surfaces of the lens and the refractive index. Neglecting the refractive index, it can be given as

\[
\frac{1}{f} = \frac{1}{R_1} - \frac{1}{R_2} \tag{3.25}
\]

where \( R_1 \) and \( R_2 \) are the radius of curvature values of the surfaces \( Z_1 \) and \( Z_2 \) respectively. Suppose that there is a certain amount of deviation at the \( Z_2 \) which can be represented by the error factor \( e_2 \). This would lead to an error factor \( e_1 \) at the \( Z_1 \) since the focal length is not changed. Then, Eq. 3.25 can be rewritten as

\[
\frac{1}{f} = \frac{1}{R_1 e_1} - \frac{1}{R_2 e_2} \tag{3.26}
\]

where factors \( e_1 \) and \( e_2 \) are given as

\[
e_1 = \frac{R_1}{R_1^i}, e_2 = \frac{R_2}{R_2^c} \tag{3.27}
\]

Here, \( R_1^i \) is the input radius of curvature value used in the ASR routine and \( R_2^c \) is the calculated value from the retrieved aspherical surface. Equating both sides of the Eq.s 3.25 and 3.26 simply leads to

\[
\frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{R_1 e_1} - \frac{1}{R_2 e_2} \tag{3.28}
\]

The goal is to find the relationship between \( e_1 \) and \( e_2 \). Therefore, rearranging the Eq. 3.28

\[
\left( \frac{1}{e_2} - 1 \right) = \left( \frac{1}{e_1} - 1 \right) \frac{R_2}{R_1} \tag{3.29}
\]
which can be simplified to

$$\varepsilon_1 = \frac{\varepsilon_2}{p}$$  (3.30)

where

$$\varepsilon_1 = \frac{R_1^c}{R_1} - 1, \varepsilon_2 = \frac{R_2^j}{R_2} - 1$$  (3.31)

and $p$ is the factor $R_2/R_1$. $\varepsilon_1$ and $\varepsilon_2$ can be interpreted as the percentage error from the actual $R_1$ and $R_2$, when they are multiplied with 100.

In order to verify the error relationship given in Eq. 3.31, simulations are performed. Errors in $\varepsilon_2$ are generated from -0.1% to 0.1% and used as an input in the aspherical surface reconstruction engine. Fig. 3.21 illustrates the calculated errors $\varepsilon_1$ in %. Moreover, different $p$ parameters are used to analyze the effect of absolute values of $R_1$ and $R_2$.

![Image of graph](image-url)

**Figure 3.21.** Effect of the error in the measurement of the second surface of the lens on the reconstructed aspherical surface. Simulations are performed for various errors in % for second surface radius of curvature $R_2$, as well as various $p$ parameter which is $R_2/R_1$.

As can be seen from Fig. 3.21, in case of $p = -1$, when there is an error in the input parameter $\varepsilon_2$ of -0.1%, the corresponding error in the calculated $\varepsilon_1$ is approximately + 0.1%. On the other side when $\varepsilon_2$ is + 0.1%, then $\varepsilon_1$ is calculated as – 0.1%. This indicates that the curvature error in the second surface is compensated by an error in the curvature of the first aspherical surface in the aspherical surface reconstruction routine. This is basically due to the fact that the best solution of the minimization process is found by introducing an error at $\Sigma_1$. It should be noted that the 0.1% of an error in $R_2$ is exaggerated in the simulations, since the
measurement accuracy of spherical surfaces using conventional methods is much better than these values.

When the change of $\varepsilon_2$ in comparison to $\varepsilon_1$ is observed, it can be seen that the relationship is approximately linear. The standard deviations, when a linear fit is performed, are well below $3\times10^{-5}$%. Hence, the simulation results agree with the Eq. 3.31.

As an example, consider an aspherical lens with $R_1 = 60$ mm and $R_2 = 300$ mm, which gives $p = 5$. Now, assume that there is a 0.01% error $\varepsilon_2$ in the measurement of $R_2$. This would lead to a 30 µm deviation from the actual surface. Using the simple relationship given in Eq. 3.31, the effect of this error in $R_1$ can be found as 0.002%, which results to 1.2 µm deviation from the actual value $R_1$.

The other two input parameters, which are used in the aspherical surface reconstruction routine, are the center thickness and the refractive index of the test lens. According to [79], typical tolerance values of the center thickness accuracy and the refractive index for high quality aspherical lenses are less than ±20 µm and $10^{-5}$ respectively. Considering these tolerance ranges, simulations are performed to investigate the effect of the deviations of these parameters from their actual values. The simulations are for $p = 1$ which is the worst case in the previous investigation. The results are presented in Fig. 3.22. Here it can be seen that the change of the error in the refractive index and the center thickness $\varepsilon_t$ show a linear relationship with the error in the calculated $R_1$. Here, center thickness error $\varepsilon_t$ is calculated using the same relationship given in Eq. 3.29 in percentage. $\varepsilon_t$ is generated in an interval of ±1%. This corresponds to an error $\varepsilon_t$ of ±0.054%, which is much less compared to the effect of $R_2$. On the other side, an error of $10^{-5}$ in the refractive index leads to an error of approximately 0.0045% error in the calculated $R_1$. Compared to the influence of the errors of the other input parameters, contribution of the error in the refractive index is negligible. It should be noted that the simulation presented above considers only homogeneous material. Inhomogeneity in the lens material would lead to different refractive index at different locations at the lens. However, this problem is generally important for molded plastic asphere because of the dramatic temperature changes during the production. For high precision glass aspheres which are produced by the conventional techniques, deviation of the refractive index inside the lens is still negligible.
Figure 3.22  Effect of the error in the input parameters a) center thickness on the calculated $R_1$  b) refractive index $n$ on the calculated $R_1$.

The summary of all considerations is: all three input parameters of the test lens used in the calculations have influences on the accuracy of aspherical reconstruction. They must all be carefully measured or at least the tolerance ranges should be known. However, it is obvious that the most influential parameter is the second surface of the aspherical lens. This surface has a dominant effect on the retrieved aspherical surface and it must be precisely measured for obtaining accurate results from the ERT method.
4. Measurements of optical performances and aspherical surface profiles

In this chapter of the thesis, experiments that have been performed with the ERT setup will be presented. Measurements are performed with four different commercial aspherical lenses. The results and discussions will be handled in the following three subsections; focal length, wavefront aberrations and surface profile measurements.

4.1. Focal length measurements

The focal length measurements are performed with the sample aspherical Lens #1. The basic idea is to characterize the focal length of the test as a function of wavelength. For this purpose, the light source is chosen as an LED with a broad spectrum, and five different bandpass filters are utilized in the setup at each measurement. These filters have central wavelengths of 470 nm, 500 nm, 532 nm, 550 nm and 580 nm. Each filter has the same spectral linewidth with FWHM of 5 nm. Then, for each case, focal length is calculated from Zernike coefficients according to Eq. 2.56. Unfortunately, there was no alternative measurement system available to compare the results of the ERT setup. Therefore, design data of Lens#1 is used and the focal length of the lens is calculated theoretically in between 450 nm and 600 nm. Fig. 4.1 illustrates the results of the measured and the simulated data.

As expected, the focal length decreases as the wavelength decreases. This is verified by both simulated and measured data. The highest deviation from the simulated data is found at 500 nm as 112 µm which corresponds to 0.28% deviation from the simulation value.

The main aspheric component of the wavefront is analyzed depending on the wavelength of the light source. Fig. 4.2 illustrates measurement and simulated results for the wavelengths 470 nm, 532 nm and 550 nm. It can be seen that measurement data shows a good fit with the simulation data for all wavelengths. Calculated P-V wavefronts and the Zernike coefficients are given in Table 4.1.
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Figure 4.1. Focal length depending on the wavelength of the light source.

Figure 4.2. Wavefront constructed with only spherical aberration term $c_{13}$ for wavelength a) 470nm b) 532nm c) 580nm.
### Table 4.1. 3\textsuperscript{rd} order spherical aberration term $c_{13}$ and P-V of the wavefront depending on wavelength.

<table>
<thead>
<tr>
<th>Wavelength (nm)</th>
<th>$c_{13}$ ($\mu$m)</th>
<th>P-V ($\mu$m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>470</td>
<td>101.04</td>
<td>151.52</td>
</tr>
<tr>
<td>500</td>
<td>99.70</td>
<td>149.55</td>
</tr>
<tr>
<td>532</td>
<td>96.83</td>
<td>145.25</td>
</tr>
<tr>
<td>550</td>
<td>96.12</td>
<td>144.18</td>
</tr>
<tr>
<td>580</td>
<td>94.68</td>
<td>142.03</td>
</tr>
</tbody>
</table>

### 4.2. Measurements of wavefront aberrations

In order to provide comparison to the results of the ERT setup, two different measurement setups are realized: Mach-Zehnder interferometer and SHS. The fundamental principles of both of the setups were described in Chap. 1. These setups are implemented in the lab environment during this thesis work. The algorithms are written in Matlab to analyze the measurement data. In the following two subsections the implemented setups will be introduced.

#### 4.2.1. Mach-Zehnder interferometer measurement setup

For measurements of wavefront aberrations of large aspherical lenses, Mach-Zehnder configuration is selected. Fig. 4.3 illustrates the schematic diagram of the measurement setup. In this setup, a HeNe laser with an output wavelength of 633 nm is used as a light source. At the output of the laser, a beam expander is located. This beam expander has a magnification factor of 16 and generates a plane wavefront with a diameter of approximately 10 mm. Laser light is splitted into two arms by the beam splitter BS1. One arm goes to the mirror M1 which is mounted on a Piezo Controller (PZT). This arm is called the reference arm. PZT can be moved in order to generate the necessary phase shift. The other arm of the interferometer, the test arm, travels the path through the mirrors M2 and M3. These two mirrors are used to align the incoming beam to the entrance of the microscope objective. This objective has a numerical aperture (NA) of 0.65, so that the generated spherical wavefront from the objective illuminates the complete aperture of the test lens. It should be noted that the radius of the spherical wavefront must be equal to the focal length of the lens under test $f_1$. This results to a quasi-plane output wavefront. In reality, this is not a perfect plane wavefront; it involves the aberrations introduced by the lens under test. A lens holder allowing the alignment in x-y-z directions is utilized. There is as well a possibility to tilt the test lens along x and y axes in order to minimize the tip-tilt misalignment errors. Output wavefront from the test lens is then
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de-magnified by a telescope consisting of the lenses L2 and L3. Obviously, this is necessary to transform the diameter of the output wavefront from test lens to the aperture of the camera. The lenses L2 and L3 are achromatic doublets and they are designed to minimize the chromatic and spherical aberrations. The lens under test in this measurement setup has a clear diameter of 40mm. The camera has 1024 pixels each having 10.8 µm pixel size. This corresponds to a sensitive area of 11.0589 mm by 11.0589 mm. Thus, de-magnification by a factor 4 is sufficient. Lenses in the telescope are therefore chosen to have \( f_2 = 300 \) mm and \( f_1 = 75 \) mm.

![Figure 4.3. Schematic diagram of the Mach-Zehnder interferometer setup.](image)

Light travelling through the reference arm, after being reflected from M1, can be aligned using the mirrors M4 and M5. Both arms of the interferometer meet at the beam splitter BS2 and are then captured by the camera. Since this interferometer has a complex setup, care has to be taken in the measurement procedure and in the alignment of the components in the setup. Following items describe the measurement procedure step by step;

1- Alignment of the beam expander at the output of the laser

2- Alignment of the M2 and M3 consecutively to the aperture of the microscope objective
3- Positioning of the lens under test so that output wavefront is approximately plane (distance to the object should be f1)

4- Positioning of the lenses L2 and L3 with respect to test lens and camera

5- Blocking the light at the reference arm, alignment of the mirrors M4 and M5 so that light hits the center of the camera

6- Unblocking the reference arm and repeat step 5 in case there is large misalignment between both arms

7- Fine adjustment of the test lens position using from it holder (x-y direction and x tilt and y tilt)

8- Shifting the M1 using the PZT generating phase shift in between the reference and the test arm

9- Capturing the interference patterns by the camera at each position of M1

The Mach-Zehnder interferometer setup is built on an optical table with a tuned damper which reduces the amplitude of mechanical vibrations. However, even with the sophisticated optical table, it was experienced that the interference patterns were very sensitive to environmental effects during the measurements. An image from the measurement setup is shown in Fig. 4.4.

![Mach-Zehnder interferometer measurement setup.](image)

For the data collection of the interferometer setup, a five step phase shifting algorithm is used. This algorithm is also known as the Hariharan algorithm [80]. This algorithm is selected since it is insensitive to PZT miscalibration and relatively robust compared to other phase shifting
algorithms [81]. This algorithm requires measurements of five interference patterns at five different locations of the M1 with a predefined spacing. Once the interference patterns are recorded, the phase difference between the reference and the test arm is calculated using Hariharan algorithm by

$$
\phi = \tan^{-1}\left( \frac{I_2 - I_4}{-1/2 I_5 + I_3 - 1/2 I_1} \right)
$$

As a consequence of the nature of the trigonometric functions, obtained phase from Eq. 4.1 has discontinuities. In the analysis algorithms, a local phase unwrapping method [82] is used to reconstruct the phase.

### 4.2.2. Shack-Hartmann wavefront sensor measurement setup

In order to measure wavefront aberrations of large diameter lenses using SHSs, the entrance pupil of the test lens should be scaled to the active area of SHS. This is basically done by implementing transformations optics. For this purpose, the test arm of the Mach-Zehnder interferometer is used. The test arm of the Mach-Zehnder interferometer is the typical reverse setup configuration (See Fig. 1.7). Using this configuration, one can measure wavefront aberrations of the test lens under identical conditions using both methods. However, the measurements can not be performed simultaneously. The light at the reference arm of the interferometer should be blocked, so that only the light from the test arm of the interferometer is incident on the SHS. Otherwise, an interference pattern would be captured by the SHS, but SHSs are not suitable for the analysis of the interference patterns.

An image from SHS wavefront measurement setup which is located at the test arm of the Mach-Zehnder interferometer is shown in Fig. 4.5. SHS implemented in the measurement setup allows 15 wavefront measurements per second at full resolution.

### 4.3. Measurement results of wavefront aberrations

Measurements of wavefront aberrations have been performed using three measurement setups: Mach-Zehnder interferometer, SHS and ERT. The test lens used in the measurements is Lens #1 which is a plano-convex aspherical lens. Measurements are performed using each method and wavefront aberrations are determined. In the analysis of the measurement data of SHS and ERT, wavefront aberrations are reconstructed by the zonal reconstruction method. In the interferometer method, wavefront is reconstructed by the local unwrapping algorithm. Then, each data set is fitted to the Zernike polynomials to represent the wavefronts.
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aberrations. The coefficients of the Zernike polynomials and RMS wavefront aberrations are determined. Throughout the analysis of the results in this section, the reconstructed measured wavefronts and their modal representations with Zernike polynomials will be denoted by $W_m$ and $W_z$ respectively. All measurements are performed at a wavelength of 633 nm. For the interferometer and the ERT setup, the diameter of the measurement field of the test aspherical lens is 32 mm. The diameter of the measurement field was limited 25 mm for SHS. This is basically due to the small active area of the SHS compared to the camera that is used in the interferometer.

![Microscope objective Test lens L2 L3 SHS](image)

**Figure 4.5.** An image from the Shack-Hartmann wavefront measurement setup. The system is implemented at the test arm of the Mach-Zehnder interferometer setup. This configuration allows measurements of a lens under identical conditions using both of these techniques.

As expected, highest spatial resolution in between the three measurement techniques is obtained from the Mach-Zehnder interferometer. The wavefront is sampled with 641 by 641 pixels. The distance between each pixel corresponds to a spatial region of 50 µm on the test aspherical lens. In the ERT setup, spacing between each scanning points are taken 250 µm. This corresponds to 129 by 129 measurement points inside the measurement field. The lowest spatial resolution was obtained from SHS. 31 by 31 wavefront data is recorded which delivers a spatial resolution of approximately 800 µm.

The design data of the Lens #1 is used to simulate the wavefront aberrations at 633 nm. This indicates the performance of the lens defined by the optical designer before the manufacturing process. The calculated RMS Zernike coefficients from each measurement method are presented in Table 4.2. The units are in waves. It should be noted that the first three Zernike coefficients are removed from the data. These coefficients indicate the constant term, tilt in x and tilt in y directions and they do not have an influence on the optical quality of the lens.
As can be seen from Table 4.2, design lens data delivers non-zero Zernike coefficients only at the $5^{th}$, $13^{th}$ and $25^{th}$ terms. This was expected, since designed aspherical lenses typically have rotationally symmetric errors such as defocus and higher orders spherical aberrations. The dominant contribution of these terms is also evident from the coefficients calculated from three measurement techniques. $c_5$ is calculated $2.181 \, \lambda$, $2.2519 \, \lambda$ and $2.1915 \, \lambda$ from interferometer, SHS and ERT setup respectively. These results are consistent for all measurement techniques and they show that measured optical performance of the lens at $\lambda=633 \, \text{nm}$ is significantly worse than its design performance. This large deviation in the defocus term can be attributed to a deviation of the radius of curvature of the manufactured asphere from its design value. The other dominant term is the spherical aberration term $c_{13}$. While design delivers $c_{13}$ of $0.1472 \, \lambda$, the measurement results are $0.436 \, \lambda$, $0.4216 \, \lambda$ and $0.3217 \, \lambda$. Deviation from the design is apparent.

Table 4.2. Calculated RMS coefficients up to 28$^{th}$ Zernike term in waves. Design and calculated coefficients from different methods are presented.

<table>
<thead>
<tr>
<th>Zernike coefficients</th>
<th>Design</th>
<th>Interferometer</th>
<th>SHS</th>
<th>ERT</th>
<th>Surface Profiler</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_4$</td>
<td>0</td>
<td>-0.057</td>
<td>0.101</td>
<td>0.0308</td>
<td>0</td>
</tr>
<tr>
<td>$c_5$</td>
<td>0.484684</td>
<td>2.181</td>
<td>2.2519</td>
<td>2.1915</td>
<td>2.205512</td>
</tr>
<tr>
<td>$c_6$</td>
<td>0</td>
<td>-0.023</td>
<td>-0.087</td>
<td>-0.0018</td>
<td>0</td>
</tr>
<tr>
<td>$c_7$</td>
<td>0</td>
<td>0.007</td>
<td>-0.003</td>
<td>0.0087</td>
<td>0</td>
</tr>
<tr>
<td>$c_8$</td>
<td>0</td>
<td>-0.097</td>
<td>-0.034</td>
<td>-0.0439</td>
<td>0</td>
</tr>
<tr>
<td>$c_9$</td>
<td>0</td>
<td>-0.079</td>
<td>-0.062</td>
<td>-0.0401</td>
<td>0</td>
</tr>
<tr>
<td>$c_{10}$</td>
<td>0</td>
<td>-0.009</td>
<td>-0.005</td>
<td>-0.0287</td>
<td>0</td>
</tr>
<tr>
<td>$c_{11}$</td>
<td>0</td>
<td>0.001</td>
<td>-0.015</td>
<td>0.0091</td>
<td>0</td>
</tr>
<tr>
<td>$c_{12}$</td>
<td>0</td>
<td>0.002</td>
<td>0.011</td>
<td>0.0045</td>
<td>0</td>
</tr>
<tr>
<td>$c_{13}$</td>
<td>0.1472</td>
<td>0.436</td>
<td>0.4216</td>
<td>0.3217</td>
<td>0.298576</td>
</tr>
<tr>
<td>$c_{14}$</td>
<td>0</td>
<td>-0.016</td>
<td>0.01</td>
<td>-0.0023</td>
<td>0</td>
</tr>
<tr>
<td>$c_{15}$</td>
<td>0</td>
<td>0.005</td>
<td>-0.001</td>
<td>0.0033</td>
<td>0</td>
</tr>
<tr>
<td>$c_{16}$</td>
<td>0</td>
<td>0.003</td>
<td>0.003</td>
<td>-0.0001</td>
<td>0</td>
</tr>
<tr>
<td>$c_{17}$</td>
<td>0</td>
<td>-0.002</td>
<td>-0.009</td>
<td>-0.0036</td>
<td>0</td>
</tr>
<tr>
<td>$c_{18}$</td>
<td>0</td>
<td>0.006</td>
<td>0.012</td>
<td>-0.0003</td>
<td>0</td>
</tr>
<tr>
<td>$c_{19}$</td>
<td>0</td>
<td>-0.018</td>
<td>-0.007</td>
<td>-0.0045</td>
<td>0</td>
</tr>
<tr>
<td>$c_{20}$</td>
<td>0</td>
<td>0.004</td>
<td>-0.005</td>
<td>0.0003</td>
<td>0</td>
</tr>
<tr>
<td>$c_{21}$</td>
<td>0</td>
<td>0.000</td>
<td>-0.002</td>
<td>-0.0002</td>
<td>0</td>
</tr>
<tr>
<td>$c_{22}$</td>
<td>0</td>
<td>0.001</td>
<td>-0.003</td>
<td>0.0004</td>
<td>0</td>
</tr>
<tr>
<td>$c_{23}$</td>
<td>0</td>
<td>0.002</td>
<td>0.003</td>
<td>-0.0008</td>
<td>0</td>
</tr>
<tr>
<td>$c_{24}$</td>
<td>0</td>
<td>0.007</td>
<td>-0.013</td>
<td>-0.0008</td>
<td>0</td>
</tr>
<tr>
<td>$c_{25}$</td>
<td>0.007597</td>
<td>0.020</td>
<td>-0.046</td>
<td>0.0141</td>
<td>-0.03737</td>
</tr>
<tr>
<td>$c_{26}$</td>
<td>0</td>
<td>-0.001</td>
<td>-0.013</td>
<td>0.0013</td>
<td>0</td>
</tr>
<tr>
<td>$c_{27}$</td>
<td>0</td>
<td>0.001</td>
<td>-0.007</td>
<td>-0.0034</td>
<td>0</td>
</tr>
<tr>
<td>$c_{28}$</td>
<td>0</td>
<td>-0.003</td>
<td>0.002</td>
<td>0.0033</td>
<td>0</td>
</tr>
</tbody>
</table>
Another important point is that $c_{13}$ obtained from interferometer and SHS are more consistent with each other compared to the result from the ERT setup. The difference is approximately 0.1 $\lambda$. The reason for this difference can be explained by the existence of the transformation optics in the setup. Mach-Zehnder interferometer and SHS are both built on the same setup using the same transformation optics. As mentioned in Chap. 1, both of the measurement systems rely on the quality of the lenses used in the measurement system. On the other side, in the ERT setup, the rays are directly traced through the test lens without any other optical component in the system. So, it can be assumed that the transformation optics additionally introduce spherical aberration to the measurements results from interferometer and SHS.

As mentioned above, 28 Zernike terms are used in the description of the measured wavefronts. This allows the representation of maximum 8th order wavefront aberrations. In order to investigate the higher frequency wavefront aberrations, the reconstructed wavefront by Zernike polynomials $W_z$ are subtracted from the measured wavefront $W_m$. The measured and fitted wavefronts and their differences for each measurement setup are depicted in Figures 4.6-4.8. Standard deviations of the differences $W_m - W_z$ are found 0.018 $\mu m$, 0.020 $\mu m$, 0.032 $\mu m$. As can be seen from Figures 4.6c, 4.7c, 4.8c, high frequency wavefront aberrations show periodic wavefront undulations which are rotationally symmetric. These errors are probably caused by the high frequency surface deformations on the aspherical surface of the lens. The wavefront undulations are almost perfectly rotationally symmetric which indicates the path of the polishing tool on the surface of the lens. When the results from the three measurement techniques are compared, a correlation between the results from interferometer and ERT is evident. SHS also delivers a similar pattern, however it clearly suffers from its poor resolution.

Calculated total RMS wavefront errors are 2.228 $\lambda$, 2.216 $\lambda$ and 2.2296 $\lambda$ for interferometer, SHS and ERT setup respectively. These results also show the agreement between the results from the three different techniques.

The summary of all the measurement results are: ERT setup for wavefront aberration measurements is verified in comparison to a Mach-Zehnder interferometer and SHS. The interferometry offers the highest spatial resolution in between these three techniques and the results from the ERT setup are comparable to interferometer. The main advantage of SHS is its high speed which almost allows instant measurement of the incident wavefront. However, its poor spatial resolution is not sufficient for high accuracy wavefront aberration measurements. ERT setup has no extra optical component in the setup and this allows direct
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wavefront measurements of the test lens. The accuracy of interferometer and SHS rely on the transformation optics used in the measurement setups.

Figure 4.6. Wavefront aberrations of Lens #1 measured by the Mach-Zehnder interferometer a) Measured wavefront $W_m$ calculated using local phase unwrapping b) Representation of the wavefront using Zernike polynomials $W_z$ c) Difference between measured and fitted wavefront $W_m - W_z$. 
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Figure 4.7. Wavefront aberrations of Lens #1 measured by the ERT setup a) Zonal reconstruction, measured wavefront $W_m$ b) Representation of the wavefront using Zernike polynomials $W_z$ c) Difference between measured and fitted wavefront $W_m - W_z$. 
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(a) 

(b) 

(c)

Figure 4.8. Wavefront aberrations of Lens #1 measured by the Shack-Hartmann wavefront sensor

(a) Zonal reconstruction, measured wavefront $W_m$
(b) Representation of the wavefront using Zernike polynomials $W_z$
(c) Difference between measured and fitted wavefront $W_m - W_z$.

4.4. Aspherical surface measurements

In order to verify the ERT method for aspherical surface measurements in practice, a commercial tactile surface profiler is used for comparison measurements. The surface profiler is the MarSurf LD120 Aspheric 3D [48]. As in all tactile measurement systems, the basic principle of MarSurf is to touch the surface under test with a stylus tip and to measure the moves of the tip while scanning the surface. For the measurements of aspherical surfaces a high precision contour instrument is combined with a rotational axis. The contour instrument
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is considered as the measurement head of the system (See Fig. 4.9). It is mounted on a pillar and it can be moved up and downwards automatically. The complete measurement system is built on top of a solid granite board which reduces effects of the vibrations. In addition, the system is located in an air-conditioned laboratory on its own base plate which is mechanically uncoupled from the floor. The whole process of calibration and the measurements are automatically controlled by the software supplied by the company.

Two different measurement modes are offered by the MahrSurf: 2D and 3D. 2D measurements are simply performed by linear scans along the center of the lens. For 3D measurements, first, two linear scans should be performed with a rotational offset of 90°. Then, concentric circular profiles are gathered to reconstruct the 3D surface profile. The device is specified by its manufacturer with vertical resolution of 2 nm and form deviations less than 100 nm. Minimum spatial resolution is specified as 50 nm.

![Measurement head](image)

**Figure 4.9.** MahrSurf LD120 Aspheric 3D. The measurement head allows linear scans on the aspherical surface under test. A rotational axis is combined to the system for 3D aspherical surface measurements.

Measurements of sample aspherical lens surfaces are performed using a stylus tip with a ruby ball (See Fig. 4.10). The diameter of the ruby ball is 500 µm. During the measurements with MahrSurf, it was found that the surface quality of the ruby ball is extremely important for the accuracy of the surface measurements [83]. First measurements were failed because of the
scratches and defects on the surface of the ruby ball. To overcome this problem, a new ruby ball was ordered and the errors were eliminated. Another important point was that the measurement system was strongly sensitive to dust and contamination on the test surface. Therefore, the test surface was carefully cleaned before each measurement.

Another problem was that, even though the system is designed for 3D measurements, it was not possible to obtain data for 3D profiles. The reason for this problem is still unknown. However, linear scans were performed successfully.

![Figure 4.10. Stylus tip with a ruby ball. The diameter of the ruby ball is 500 µm.](image)

### 4.4.1. Lens #1

The first test lens, Lens#1, is a commercial plano-convex aspherical lens. Aspherical surface of the lens is measured by the ERT setup and the MahrSurf. In the following discussions about the measurement results MahrSurf will be denoted as the “Surface Profiler”.

In the ASR routine, the refractive index of the lens at $\lambda=633$ nm is used as an input parameter. The second surface is assumed to be a perfect plane surface. The measurements are performed in a diameter 40 mm. Distance between each scanning point at the ERT setup was set to 100 µm, which gives 401 points across the linear scan. On the other side, it was possible to obtain approximately 15456 points in the same measurement interval. The raw measurement data from the Surface Profiler is filtered with a 0.2 µm low pass filter. This filtering is necessary since raw surface data includes significant high frequency noise.

The measured surface data from both systems are analyzed by the following three steps:
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1 – Deviation between the measured and the design aspherical surface $Z_a - Z_d$. This can be regarded as the global deviation from the theoretical form of the aspherical surface.

2 – Deviation between the measured and the best-fit radius aspherical surface $Z_a - Z_f(R_f)$. Here only the radius of curvature $R$ is varied in the fitting process. The conic constant $k$ and the aspherical coefficients $A$ are same as the design parameters.

3 – Deviation between the measured and design surface after fitting the best-fit radius and the aspherical terms. $Z_a - Z_f(R_f, A_f)$. The conic constant $k$ is same as the design parameter.

The deviation between the measured and the design aspherical surface profiles from the ERT setup and the Surface Profiler are illustrated in Fig. 4.11. As can be seen, the results from both measurement systems indicate a significant departure from the design surface, which includes a strong low frequency component. P-V surface deviation for the ERT setup and Surface Profiler are measured as 3.018 µm and 2.033 µm, respectively.

![Figure 4.11. Deviation of the measured aspherical profile $Z_a$ from the design aspherical profile $Z_d$ – Lens#1.](image)

For the further analysis of the measurement data, the calculated fit coefficients from the ERT setup and the Surface Profiler are presented in Table 4.3. It can be seen that the best-fit radius value calculated from the ERT setup and the Surface Profiler are 32.5197 mm and 32.519 mm, respectively. The deviations from the best-fit radius aspherical surfaces are shown in Fig. 4.12. In this case, P-V deviation for the ERT setup is found as 1.2 µm and Surface Profiler gives P-V deviation of 0.9 µm. This difference can be explained by the mismatch of
the results at the both edges of the lens. However, overall shape of the deviations shows good agreement with each other (See Fig. 4.12).

When the measured surface data is fitted with the radius of curvature and high order aspherical term, it can be seen that low frequency components of the surface deviations are filtered by the fitted aspheric coefficients. The residuals seen in Fig. 4.13 indicate surface deformations with various spatial periods from 4 mm to 10 mm. Rotational symmetric periodic oscillations can be clearly seen. The basic waviness of the surface deformations with varying amplitudes from 100 nm to 280 nm shows a great correlation. P-V surface deformations for the ERT setup and the Surface Profiler are calculated as 574 nm and 555 nm. For the same surface data, RMS surface deformations are found 88.3 nm and 91.2 nm.

Table 4.3. Comparison of the aspherical surface measurement results between the ERT setup and the Surface Profiler for Lens #1. Both measurement data sets are analyzed by fitting to best-fit radius and fitting to all parameters. Calculated fit coefficients are indicated with bold numbers.

<table>
<thead>
<tr>
<th></th>
<th>$Z_d$</th>
<th>$Z_f(R_f)$</th>
<th>$Z_f(R_f)$</th>
<th>$Z_f(R_f, A_f)$</th>
<th>$Z_f(R_f, A_f)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ERT</td>
<td>32.5</td>
<td>32.5197</td>
<td>32.5190</td>
<td>32.5323</td>
<td>32.5096</td>
</tr>
<tr>
<td>$k$</td>
<td>-1.71664</td>
<td>-1.71664</td>
<td>-1.71664</td>
<td>-1.71664</td>
<td>-1.71664</td>
</tr>
<tr>
<td>$A_2$</td>
<td>4.10114e-06</td>
<td>4.10114e-06</td>
<td>4.21213e-06</td>
<td>4.31885e-06</td>
<td></td>
</tr>
<tr>
<td>$A_6$</td>
<td>1.32564e-13</td>
<td>1.32564e-13</td>
<td>1.32564e-13</td>
<td>1.32564e-13</td>
<td>1.32564e-13</td>
</tr>
<tr>
<td>$A_8$</td>
<td>-6.4285e-17</td>
<td>-6.4285e-17</td>
<td>-6.4285e-17</td>
<td>-6.4285e-17</td>
<td>-6.4285e-17</td>
</tr>
</tbody>
</table>

An interesting point is the difference between the calculated coefficients for $Z_f(R_f, A_f)$ from the ERT setup and the Surface Profiler surface data. As can be seen from Table 4.3, the difference between calculated $R$ is approximately 23 µm. However, the reconstructed deviations from both methods are almost identical as in Fig. 4.13. It can be stated that the deviation in the $R$ is compensated by the aspherical coefficients. This outcome was actually expected, since the aspherical surface function is not orthogonal. Furthermore, the design aspherical surface of this lens includes the second order term $A_2$ which directly interacts with the radius of curvature $R$. Therefore, the values of the coefficients might be misleading when comparing two different measurement data sets.
Measurements of optical performances and aspherical surface profiles

Figure 4.12 Deviation of the measured aspherical profile $Z_a$ from the best-fit radius aspherical surface $Z_f(R_f)$. Aspherical coefficients of $Z_f(R_f)$ are same as the design surface $Z_d$ – Lens#1.

Figure 4.13 Deviation of the measured aspherical profile $Z_a$ from the fit aspherical profile $Z_f(R_f, A_f)$ – Lens #1. 3D surface deformations of the Lens #1 are measured using the meander scan geometry by the ERT setup. The retrieved 3D surface data is subtracted from the best-fit radius aspherical surface and the result is illustrated in Fig. 4.14. P-V and RMS surface deformations are calculated as 804 nm and 123.6 nm, respectively. The surface deformations that are seen in Fig. 4.14 indicate rotational symmetric errors fluctuating between 228 nm and -576 nm. An
important outcome from these results is that the effects of the polishing machine on the surface of the lens are clearly detectable. Not only the surface undulations with respect to the radial coordinate, but also periodic azimuthal surface deformations are evident. These results also correlate with the high frequency wavefront aberrations measured by the interferometer and the ERT setup (See Fig. 4.6c and Fig.4.7c).

\[Figure 4.14.\] Deviation of the measured 3D aspherical profile \( Z_a \) from the best-fit radius aspherical surface \( Z_f(R_f) \). Aspherical coefficients of \( Z_f(R_f) \) are same as the design surface \( Z_d \) – Lens#1.

### 4.4.2. Lens #2

Lens #2 is also a plano-convex aspherical lens. The same analysis procedure as in Lens #1 is applied to the measurement data from the ERT setup and the Surface Profiler. The theoretical surface of this lens is specified by up to 16\(^{th}\) order aspherical term. The design and the calculated aspherical surface coefficients are presented in Table 4.4.

The departures of the measured aspherical profiles from the design aspherical profile are depicted in Fig. 4.15. Similar to Lens#1, a significant departure can also be seen which is basically because of the deviation in \( R \) from its theoretical value. When the best-fit radius value is calculated, P-V of \( Z_a-Z_f(R_f) \) are found as 364 nm for the ERT setup and 272 nm for the Surface Profiler, respectively (See Fig. 4.16). Furthermore, calculated RMS of \( Z_a-Z_f(R_f) \) are 63.8 nm and 78.6 nm. The deviation between the calculated best-fit radius values from both measurement systems is 1.9 \( \mu \)m. There might be a couple of possible reasons for this difference. As discussed in Sec. 3.5.8, a deviation of the refractive index of the lens from its
theoretical value could generate such an error in the calculated best-fit radius values. Another possibility is that linear scans are not performed exactly on the same regions of the test lens.

Table 4.4. Comparison of the aspherical surface measurement results between the ERT setup and the Surface Profiler for Lens#2. Both measurement data set are analysed by fitting to best-fit radius and fitting to all parameters. Calculated fit coefficients are indicated with bold numbers.

<table>
<thead>
<tr>
<th></th>
<th>$Z_d$</th>
<th>$Z(R_f)$</th>
<th>$Z(R_f)$</th>
<th>$Z(R_f, A_f)$</th>
<th>$Z(R_f, A_f)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ERT</td>
<td>Surface Profiler</td>
<td>ERT</td>
<td>Surface Profiler</td>
<td></td>
</tr>
<tr>
<td>$R$</td>
<td>31.075</td>
<td>31.0866</td>
<td>31.085</td>
<td>31.0911</td>
<td>31.0869</td>
</tr>
<tr>
<td>$k$</td>
<td>-0.744</td>
<td>-0.744</td>
<td>-0.744</td>
<td>-0.744</td>
<td>-0.744</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$A_6$</td>
<td>-2.27135e-10</td>
<td>-2.27135e-10</td>
<td>-2.27135e-10</td>
<td>-2.08027e-09</td>
<td>1.12187e-10</td>
</tr>
<tr>
<td>$A_8$</td>
<td>-1.70421e-13</td>
<td>-1.70421e-13</td>
<td>-1.70421e-13</td>
<td>7.89594e-12</td>
<td>-6.67895e-12</td>
</tr>
<tr>
<td>$A_{10}$</td>
<td>-3.68093e-17</td>
<td>-3.68093e-17</td>
<td>-3.68093e-17</td>
<td>-5.26350e-15</td>
<td>-1.13374e-16</td>
</tr>
<tr>
<td>$A_{12}$</td>
<td>8.94434e-17</td>
<td>8.94434e-17</td>
<td>8.94434e-17</td>
<td>-4.74362e-17</td>
<td>-1.14265e-19</td>
</tr>
<tr>
<td>$A_{14}$</td>
<td>1.85011e-23</td>
<td>1.85011e-23</td>
<td>1.85011e-23</td>
<td>8.614789e-20</td>
<td>-1.14265e-19</td>
</tr>
</tbody>
</table>

When all the aspherical terms are also used in the fitting process, it can be seen from Fig. 4.17 that the low frequency components are completely filtered. The residuals are the high frequency surface deformations. P-V surface deformations of $Z_a - Z(R_f, A_f)$ are 204.6 nm and 197.9 for the ERT setup and the Surface Profiler, respectively. In the same order, RMS of $Z_a - Z(R_f, A_f)$ are calculated as 35.6 nm and 33.4 nm. A good agreement between the results of the two techniques is obvious. In comparison to Lens#1, the frequency of the waviness of the surface of Lens#2 is higher and amplitudes of the periodic oscillations are lower. According to Küchel’s classification of aspherical surfaces given in Sec. 1.1.1, both lenses are regarded as precision aspheres. However, surface quality of Lens#2 is higher that of Lens#1 and it is closer to be a high precision asphere.
Figure 4.15. Deviation of the measured aspherical profile $Z_a$ from the design aspherical profile $Z_d$ – Lens#2.

Figure 4.16. Deviation of the measured aspherical profile $Z_a$ from the best-fit radius aspherical surface $Z_f(R_f)$. Aspherical coefficients of $Z_f(R_f)$ are same as the design surface $Z_d$ – Lens#2.
In order to quantify the repeatability of the ERT setup, 15 measurements are performed consecutively. The lens has kept at the same orientation during the measurements and the environmental conditions were identical. For each measurement data set, best-fit radius \( R_f \) is calculated and surface deformations \( Z_a - Z_f(R_f) \) are determined. Fig. 4.18 shows the results of mean value of 15 measurement results as well as the corresponding standard deviation at each yw-position. The maximum value of the standard deviations is found as 3.1 nm. It can be seen that calculated standard deviations are slightly increased at the edges of the aperture. Furthermore, the mean value of the standard deviations for all y-positions is calculated as 1.9 nm. Moreover, the standard deviation of the calculated best-fit radius values for 15 measurements is found as 79.8 nm. It can be concluded that high repeatability has been achieved by the setup.
Measurements of optical performances and aspherical surface profiles

4.4.3. Lens #3

Lens#3 is a convex-convex commercial aspherical lens. The conic constant and the aspherical coefficients of this lens are kept confidential by the manufacturer. Here, only the radius of curvatures of the two surfaces will be given.

The radius of curvature $R$ of the second surface $Z_2$ of the Lens#3 is measured with the Surface Profiler and it is found as 599.958 mm. This value is used as an input parameter to the aspherical surface reconstruction routine as well the theoretical values of the thickness and the refractive index of the lens.

The calculated best-fit radius values of the measured aspherical profiles are given in Table 4.5. A noticeable deviation around 850 µm from the theoretical value of $R$ is apparent. On the other hand, the difference of the fitted $R$ between two techniques is 23 µm. This difference is relatively larger compared to the results of the plano-convex Lens#1 and Lens#2.

As mentioned above, unlike the plano-convex aspherical lenses, the accuracy of the measurement of the radius of curvature of the second surface of the lens plays a critical role in the ASR routine. The effect of the error in the second surface on the calculated radius of curvature of the aspherical surface was given in Eq. 3.30. According to this equation, the factor $\rho$ for this lens is 5.7. Assuming a 0.1% measurement error in the radius of curvature of
the second surface, the error in the radius of curvature of the aspherical side of the lens is 0.018%. This would lead to an error of 18 µm in $R$ of the asphere. Moreover, the effects of the thickness and the refractive index errors, as well as the mismatch of the linear scan regions further contributes to the error in the calculated $R$.

Table 4.5. Design and the measured radius of curvatures of the first and second surface of the Lens#3.

<table>
<thead>
<tr>
<th>$Z_{1a}$</th>
<th>$Z_{1b}$</th>
<th>$Z_{2a}$</th>
<th>$Z_{2b}$</th>
<th>$Z_{i}(R_f)$</th>
<th>$Z_f(R_f)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface Profiler</td>
<td>ERT Surface Profiler</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R$</td>
<td>105.367</td>
<td>601</td>
<td><strong>599.958</strong></td>
<td>106.215</td>
<td>106.238</td>
</tr>
</tbody>
</table>

When the deviations $Z_a - Z_f(R_f)$ are observed (See Fig. 4.19), the results from both techniques show good correlation. P-wV values of the residuals are found as 259 nm and 210 nm for the ERT setup and the Surface Profiler. The corresponding RMS values are 75 nm and 82 nm.

A remarkable outcome from both of the measurement techniques is the asymmetric shape of the residuals $Z_a - Z_f(R_f)$. The relative peak around region $y=-15$ mm has its symmetry around $y=+15$ mm. However, the peak in the region $y=-2$ mm generates an asymmetric geometry. In order to analyze this error, 3D measurements of the surface with the ERT setup are performed. The main idea is to repeat the 3D measurements at two different orientation of the test lens. The relative change in the orientation is $90^\circ$ along the $z$-axis. The results of both measurements are depicted in Fig. 4.20. Here, the red regions clearly indicate the non-
rotational asymmetric residuals on the surface of the lens. When the lens is rotated with 90°, it can be seen that the asymmetric pattern rotates as well with 90°. This indicates an error in the local correction process of the surface of the aspherical lens.

![Graphical representation](image)

**Figure 4.20.** 3D surface measurements of Lens#3 have been performed at two locations of the lens. **a)** Orientation-1 0° **b)** Orientation-2 rotated by 90° around z-axis.
4.4.4. Lens #4

Lens #4 is a convex-convex commercial aspherical lens. It has the same design parameters of Lens #3. The radius of curvature $R$ of the second surface $Z_2$ of the Lens #4 is measured with the Surface Profiler and it is found as 598.847 mm. The calculated best-fit radius values of the measured aspherical profiles are given in Table 4.6. Compared to that of Lens #3, the departure of the fitted $R$ from the design $R$ is less, approximately 250 µm. However, the difference between the fitted radiuses of curvatures between two techniques is 28 µm, which is quite close to Lens #3. Again, it is expected that an error in the measurement of the radius of curvature of the second surface is the dominant contributor of this difference.

Table 4.6. Design and the measured radius of curvature of the first and second surface of the Lens #3.

<table>
<thead>
<tr>
<th></th>
<th>$Z_{1_d}$</th>
<th>$Z_{2_d}$</th>
<th>$Z_2$</th>
<th>$Z_f(R_f)$</th>
<th>$Z_f(R_f)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface Profiler</td>
<td>105.367</td>
<td>601</td>
<td>598.847</td>
<td>105.647</td>
<td>105.619</td>
</tr>
<tr>
<td>ERT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The calculated deviations $Z_a - Z_f(R_f)$ are depicted in Fig. 4.20. P-V of 143 nm and 144 nm are obtained from the ERT setup and the Surface Profiler. The corresponding RMS values are 35 nm and 26 nm. Furthermore, as can be seen in Fig. 4.21, the forms of the residuals of the aspherical surface measurements from both methods are in good agreement.

Figure 4.21. Comparison of the measurement results of lens #4 by the ERT setup and the Surface Profiler. Line scan measurements are performed and the measured surface is subtracted from the best-fit radius aspherical surface.
As mentioned in the Sec. 1.1.1, determination of the aspherical surface slope errors is important for characterization of aspherical lenses. As a next step, surface slope errors of the Lens#4 are measured. For the surface slope error calculations, Eq. 1.5 is used and the Integration Length is taken as 100 µm. The maximum slope error is found as 108.74 µrad and the RMS slope error is calculated as 17.87 µrad. The Surface Profiler is not capable of measuring the surface slope errors; therefore no comparison measurement was possible. The results from the ERT setup are shown in Fig. 4.22. Here, both surface deformations and the slope errors are plotted in the same figure. It can be clearly observed that the surface slope errors show the same tendency as the surface deformations. They are both rotational symmetric. Especially for the spatial regions $y = \pm 3$ mm, the surface slope error increases and exceeds 100 µrad. On the other side, for the same region, surface departure is well below 50 nm. This indicates that analyzing only the aspherical surface departure might be misleading. Particularly for high and ultra precision aspheres, surface slope errors should also be quantified during the manufacturing process.

![Figure 4.22. Surface deformations and surface slope error of lens the #4 measured by the ERT setup.](image-url)
5. **Summary**

This thesis was motivated by the increasing demand in the aspherical lens manufacturing industry for flexible and accurate measurement systems. The long standing goal of non-contact measurements of aspherical lenses without the use of a reference object (Null optics or CGH) is still not achieved. Thus, the potential of the ERT method for aspherical lens testing has been investigated.

In the ERT method, the slopes of the transmitted rays through the test lens are determined by capturing the ray positions along the propagation direction of the ray. The required information about the test lens is calculated from these slopes. The most important feature that gives the flexibility to ERT method is the \( z \) positioning of the camera. The \( z \) positions of the camera can simply be programmed by the control unit software. This feature makes the dynamic range and wavefront sensitivity of the ERT method adjustable. It has been proven that by increasing the distance \( \Delta z \) between two \( z \)-positions of the camera, slope uncertainty in the region of several \( \mu \text{rad} \) can be achieved. In addition, sequentially scanning the surface of the test aspherical lens allows high dynamic range measurement. By this way, the problem of overlapped spots, which limits the dynamic range of SHS, is eliminated. It is calculated that an aspherical wavefront with a P-V of 1.561 mm can be measured by the ERT method.

In order to compare the optical performance measurements of the ERT setup with established methods, a Mach-Zehnder interferometer and a SHS measurement setup has been realized. The differences between the calculated total RMS wavefront errors for a test lens from these three setups were less than 0.01 \( \lambda \). Furthermore, high correlation between the results of the ERT setup and the interferometer regarding the high frequency wavefront aberrations has been observed.

In this thesis work, a new approach called “aspherical surface retrieval” has been proposed. The main principle is to use slopes of the transmitted rays for calculating the aspherical surface of the lens. This method includes two different analyses for two different types of aspherical lenses. For plano-convex aspherical lenses, direct relationship between the slopes of the transmitted rays and the aspherical surface of the lens has been derived. The
assumption is that the second surface of the lens and the refractive index of the lens at the test wavelength are known. The measurement results from ERT and a commercial profiler showed a great agreement for two different commercial lenses. The differences of the RMS values of the surface deformations and calculated best-fit radius values were less than 15 nm and 2 µm respectively.

For aspherical lenses having non-planar second surfaces, the retrieval of the aspherical surface profile is done using an optimization process between the actual lens and a model lens. The second surface of the lens has to be known and this information is used as an input parameter in the ASR routine. The systematic error sources of the ASR approach are the deviations of the refractive index, thickness and the second surface from their actual values. It is calculated that if the deviation of the thickness and refractive index of the lens is in the typical tolerance range of precision aspherical lenses, effects of these systematic errors are not significant. It has been verified that the most influential parameter is the measured radius of curvature of the second surface. An error in this parameter has simply an additive effect on the error in the calculated radius of curvature of the aspherical first surface. Measurement results of two different aspherical lenses using ERT and the surface profiler validated this argument. A clear correlation between the surface deformations measured by both methods has been shown. However, up to a deviation of 27 µm in the calculated best-fit radius value of the aspherical surface was obtained.

A notable outcome from the simulations of the ASR approach was the difficulty of the representation of the high frequency surface deformations using the conventional aspherical surface equation. Using maximum 20th order aspherical term, it was possible to retrieve surface deformation up to a spatial frequency of 3 mm⁻¹. Theoretically, infinite number of aspherical terms can be used in the optimization process. However, it is practically problematic. For this reason, recently proposed Forbes polynomials can be used as an alternative to represent the aspherical surface. According to [84], Forbes polynomials can represent the same aspherical surface using less aspherical terms compared to the conventional aspherical surface equation. Furthermore, these polynomials have the advantage to be orthogonal. It should be noted that, for implementation of these polynomials to the ASR routine, numerical ray tracing algorithms presented in Chap. 2 shall be adapted to the Forbes polynomials.

Finally, it can be concluded that the ERT method can be used in simultaneous measurements of the optical performances and the surface profiles of the aspherical lenses. In addition, there
is no requirement of a reference object in the measurement setup. As a further investigation, the ERT setup can be implemented to the classical fabrication process of the aspherical lenses. The measurement results of the ERT setup should be imported to the instruments used in the polishing and surface finishing steps. By this way, the deviations of the aspherical surface from its design form can be corrected. Moreover, it would be also possible to monitor the optical performance of the lens already during the surface fabrication process.
Appendix A

Flowchart of the control unit

Start -> Initialization
- XY stage
- Z-stage
- Camera

Calibration?
- yes -> Calibration
- no -> Input Parameters
- Wavelength
- Diameter of test lens
- Scan type
- No. of scan points
- Z positions
- Axes speed

Scan Type = Meander Scan

For every Z position
- Drive to Z Position
- Drive to Start XY Position
- Start Scanning

For every X or Y position
- Wait for trigger signal
- Grab image
- Calculate Control XY
- Write into File

X or Y pos = last X or Y?
- yes -> Z pos = last Z?
- no

Z pos = last Z?
- yes
- no

Scan Type = Line Scan

X or Y

Calibration?
- no -> Calibration
- yes -> Initialization
- XY stage
- Z-stage
- Camera

For every Y position
- Wait for trigger signal
- Grab image
- Calculate Control XY
- Write into File

X Pos = last X
- yes -> Y Pos = last Y
- no

Y Pos = last Y
- yes -> Drive to Y:
- no

Drive to Y:
- yes
- no

Close

for every Z position
- Drive to Z Position
- Drive to Start XY Position
- Start Scanning

for every X Position
- Wait for trigger signal
- Grab image
- Calculate Control XY
- Write into File

X Pos = last X
- yes
- no

Y Pos = last Y
- yes
- no

Z pos = last Z?
- yes
- no

Close
Appendix B

Flowchart of the analysis software
Publications


Bibliography


